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Solving Intuitionistic Fuzzy Unconstrained Optimization Problems Using Interval Newton's Method

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Abstract:

This research work studies the optimization of intuitionistic fuzzy valued functions in unconstrained problems. The Interval Newton's Method using an Intuitionistic approach addresses both single and multivariable optimization problems. The study incorporates a mathematical comprehension of interval intuitionistic fuzzy valued problems, as well as real-world examples to demonstrate their effectiveness. Furthermore, MATLAB code is provided to demonstrate the implementation of the Interval Newton's Method.

Keywords: Unconstrained Optimization Problems, Interval Newton's Method(INM), intuitionistic fuzzy numbers.

1. Introduction

This paper focuses on unconstrained optimization problems involving Intuitionistic Fuzzy-valued functions. It delves into both single-variable and multi-variable scenarios, providing explanations and illustrations to demonstrate the application of the Interval Newton Method in tackling these optimization problems. Helmut Ratschek[4]explored interval methods in global optimization, providing solutions for diverse optimization scenarios, including unconstrained, constrained, and nonsmooth optimization. It underscores the importance of bisection techniques in interval-based global optimization algorithms, where the problem domain is recursively divided and also acknowledges advancements in bisection strategies over the last decade. Furthermore, it emphasized the significance of interval arithmetic in handling global optimization problems, as it offers global information compared to local optimization methods that rely on limited, local data. Recently, we have had to optimize provided systems and deal with linear and NLPP. The phrase optimization refers to reaching the best possible outcome.

The interval analysis approach emerged between 1950 and 1960. Interval analysis, introduced with the development of computational mathematics, is a method of describing by substituting a single fixed point with an interval. In general, we tackle nonlinear programming problems that require only crisp integers in the goal and a limited co-efficient. When dealing with such natural situations, challenges exist because of uncertainty and inexactness in the derived parameters. Since occurrence of the uncertainty and inexactness, it is inadequate to apply the classic method to handle such problems. Karl Nickel [6]explained the application of the Newton technique and interval arithmetic in solving n-dimensional vector problems. It delves into the characteristics of the Newton operator, conditions for solution existence, and limitations on error tolerances for convergent sequences. The concept is further illustrated through an example in convex programming, showcasing its real-world implications. Ved

P. Madan [15]The Interval Newton Method, especially the Hansen-Greenberg Approach, is a technique for solving systems of nonlinear equations by linearizing the system and making different changes to enhance efficiency. This approach uses preconditioning, the Hansen-Sengupta step, and interval iterations to provide more accurate and efficient solutions for solving complicated mathematical problems. The paper's proposed procedural enhancements attempt to increase algorithm performance by adding improved approximations, a sequential overrelaxation methodology, and interval iterations as alternatives to existing methods. The discussed Interval Newton Method aims to efficiently solve systems of nonlinear equations by integrating three subalgorithms: a Gauss-Seidel-type step, a real Newton iteration, and linearized equation solving via elimination. This approach improves upon existing techniques like Krawczyk's method, offering better solution efficiency.

The Newton method is a numerical method for solving equations that starts with an initial guess and then uses iterative steps to converge towards the true solution. It is based on the idea of approximating a function as a straight line tangent to the curve at a given point. To implement the interval Newton method, we will make use of interval arithmetic, where the result of an operation on intervals is an interval that bounds the possible values of the output. This allows to compute rigorous bounds on the solution of an equation. In this implementation, the `f` and `df` arguments are functions that represent the original function and its derivative, respectively. The `x0` argument is the initial guess for the root of the equation. The `epsilon` argument specifies the desired precision of the solution. The `interval` function is used to compute an interval from a single value. The method iteratively updates the value of `x` using the interval Newton iteration formula until the absolute value of `f(x)` is less than `epsilon`, indicating a suitable approximation has been found. The `df interval` is used to compute the slope of the tangent line at each iteration. The impact of employing a local minimizer to update the upper bound on the global minimum is also considered. This approach, named LISS_LP, is demonstrated through various global optimization problems, particularly focusing on those originating from chemical engineering. LISS_LP proves effective in solving problems with a vast number of local optima and those involving a relatively large number of variables. In 2017, Lei Wang described the Newton iteration-based interval uncertainty analysis approach for interdisciplinary systems, known as NI-IUAM, this technique divides complicated systems into distinct disciplines and uses Newton iteration equations to calculate the upper and lower limits of connected state variables at each iterative step. NI-IUAM requires simply the boundaries of unknown parameters, removing the requirement for particular distribution formats and perhaps reducing the need for raw data. Additionally, NI-IUAM benefits from Newton iteration's super-linear convergence, which accelerates the convergence process.

In 2021, Zhiping focused on studying the impact of uncertainties in nonlinear structural system, a Newton iteration-based interval analysis method (NI-IAM) to quantify these uncertainties by considering uncertain-but-bounded parameters as interval variables. The method simplifies the uncertain onlinear problem into a series of uncertain linear issues through interval iteration schemes, TaylorSeriesExpansion, and the LagrangianMultiplierMethod. Numerical examples demonstrate the effectiveness and applicability of the proposed method, and its compatibility with the perturbation-based probabilistic method is also studied.

Fuzzy Sets, initially introduced by Zadeh in 1965, have significantly impacted various fields due to their ability to represent the ambiguity in real-world scenarios. Triangular Fuzzy Numbers, a crucial aspect of fuzzy set theory, have been discussed in detail, along with their operations. Later, Krassimir Atanassov introduced Intuitionistic Fuzzy Sets in 1983, further expanding upon Zadeh's foundation. In addition to degrees of membership, intuitionistic fuzzy sets also incorporate degrees of non-membership. This means that for each element, there are not only measures of how much it belongs to the set, but also measures of how much it does not belong to the set.

This dual perspective offers a more comprehensive representation of uncertainty and ambiguity in real-world situations. Intuitionistic fuzzy sets have found applications in various fields, including decision-making, pattern recognition, image processing, medical diagnosis, and more. They provide a powerful framework for modeling and reasoning with uncertain and imprecise information, offering more nuanced solutions to problems where traditional crisp sets or classical fuzzy sets may fall short. Throughout the last few decades, various academics have created optimization problems with fuzzy valued objective issues. Ganesan et al. (2017) presented a comparable solution to fuzzy unconstrained optimization problems using triangular fuzzy numbers (2020).

Throughout the last few decades, different researchers have created optimization problems with fuzzy valued objective problems. [Ben et al. [1]. Debski [2], Dennis and Shanable [3], Cheng and Kovalyov [10], Timothy [11], Shi [21]. M Furthermore, unconstrained problems are solved using differential calculus. Ganesan et al.,[12] propose a novel methodology for addressing single-variable problems without constraints using Newton's interval analysis method, which differs from the standard method of employing calculus.. Sophia porchelvi et al[8] extended to bivariate problems using Newton's method. Irene hepzibah,Shilpa[5] extended into fuzzy single and bivariate unconstrained problems by using fuzzy Newton's interval analysis method. This paper presents a novel approach to solving non-linear, unconstrained optimization problems using the Intuitionistic Newton Method and interval analysis. It introduces interval arithmetic operations for addressing intuitionistic fuzzy unconstrained optimization issues. Section 3 focuses on applying Newton's method and Interval Newton's method for unconstrained optimization, while section 4 provides illustrative examples. The study concludes with some remarks in section 5.

2. Preliminariesl

In this section, some important definitions are provided

Definition 1: [13]:

A fuzzy set Ã can be expressed as the pair $x, \mu_{\tilde{A}}(x)$, where x is an element from a classical set A and $\mu_{\tilde{A}}(x)$ is the membership function associated with x. This function takes values within the interval [0, 1] and indicates x's degree of membership in the fuzzy set A.The fuzzy set Ã can be symbolized by the notation $\{x, \mu_{\tilde{A}}(x): x \in \mu_{\tilde{A}}(x) \in [0,1]\}$. This notation emphasizes the link between the classical set A and the membership function $\mu_{\tilde{A}}(x)$.

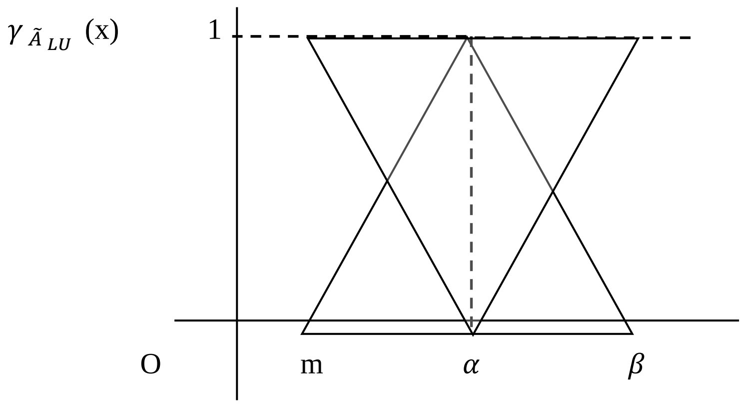
Definition 2:

L-U Parametric Representation of Intuitionistic Triangular Fuzzy Number

A Intuitionistic triangular fuzzy number $\tilde{A}^{I}{}_{LU} = (m, \alpha, \beta);\ (m', \alpha', \beta')$ where $m = b, \alpha = b - a \geq 0, \beta = c - b \geq 0. m' = b', \alpha' = b' - a' \geq 0, \beta' = c' - b' \geq 0.$

$$\mu_{\tilde{A}_{LR}}(x) = \begin{cases} 0 & ; -\infty < x \leq m \propto \\ 1 - \frac{m-x}{\alpha} & ; m - d \leq x < m \\ 1 - \frac{x-m}{\beta} & ; m \leq x < x + \beta \\ 0 & ; m + \beta \leq x < \infty \end{cases}$$

$$\gamma_{\tilde{A}_{LU}}(x) = \begin{cases} 1 & ; -\infty < x \leq m \propto \\ \frac{m-x}{\alpha} & ; m - d \leq x < m \\ \frac{x-m}{\beta} & ; m \leq x < x + \beta \\ 1 & ; m + \beta \leq x < \infty \end{cases}$$



DEFINITION 3:

RANKING FUNCTION

An efficient method for computing Intuitionistic Fuzzy numbers involves the use of a ranking function R, which maps Intuitionistic Fuzzy numbers (f) to the real line (R) ,$\mathfrak{R}$:f(R)→R. This function enables the establishment of a natural order within the set F(R) of Intuitionistic Fuzzy numbers defined on the real number set. With this ranking function, comparisons and ordering of Intuiti0nistic Fuzzy numbers can be effectively performed.

R(A) = m+[$(\beta - \alpha)$]/4+($4m' + (\beta' - \alpha'))/4$)

2.2 Arithmetic operation for Triangular intuitionistic fuzzy Numbers :

Let $\tilde{A}^{I} = \{(a_1, a_2, a_3); (a_{1'}, a_2{}', a_{3'})\}$ and $\tilde{B}^{I} = \{(b_1, b_2, b_3); (b_{1'}, b_2\ , b_{3'})\}$ are two TIFN ,

Addition:

$\tilde{A}^{I} + \tilde{B}^{I} = \{a_1 + b_1, a_2 + b_2, a_3 + b_3); (\ a_{1'} + b_{1'}, a_2{}' + b_2{}', a_{3'} + b_{3'})$ is also a TIFN

Subtraction

$\tilde{A}^I - \tilde{B}^I = \{a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_{1'} - b_{3'}, a_2{}' - b_2{}', a_{3'} - b_{1'})$ is also a TIFN

Multiplication

$\tilde{A}^I.\tilde{B}^I = \left(a_1.R\left(\tilde{B}^I\right), a_2.R\left(\tilde{B}^I\right), a_3.R(\tilde{B}^I)\right); (a_{1'}.R(\tilde{B}^I), a_2{}'.R(\tilde{B}^I), a_{3'}.R(\tilde{B}^I)\big)$ is also a TIFN, where $R\left(\tilde{B}^I\right) = (a_1 + [(a_2 - a_1)]/4) + (4a_1{}' + (a_2{}' - a_3{}'))/4)$

Division

$\tilde{A}^I/\tilde{B}^I = \left(a_1/R\left(\tilde{B}^I\right), a_2/R\left(\tilde{B}^I\right), a_3/R(\tilde{B}^I)\right); (a_{1'}/R(\tilde{B}^I), a_2\,/R(\tilde{B}^I), a_{3'}/R(\tilde{B}^I)\big)$ is also a TIFN, where $R\left(\tilde{B}^I\right) = (a_1 + [(a_2 - a_1)]/4) + (4a_1{}' + (a_2{}' - a_3{}'))/4)$

2.3 INTERVAL VALUED INTUITI0NISTIC FUZZY NUMBER

A Interval valued Intuitionistic fuzzy number $\tilde{u}^I$ is pair of $\left[\underline{u}, \bar{u}\ \right]$ of $\underline{u}(r), \bar{u}(r);\ 0 \le r \le 1$ which satisfies the following conditions: Stefen et al enhancing the efficiency of a nonlinear-system-solver using a Componentwise Newton Meth0d, introduce improvements to the method, such as utilizing index-lists, combining with an Interval Newton Gauss-Seidel Step, and verifying solution uniqueness and it provides algorithmic descriptions, properties of the method, examples, and results to demonstrate the effectiveness of the proposed approach in solving nonlinear equations

Let $\tilde{v} = \left[\underline{v}(r), \bar{v}(r)\right]$

$x > 0; x = [x\underline{v}(r), x\bar{v}(r)]$ and $x < 0; x = [\ x\bar{v}(r), x\underline{v}(r)]$

$$\tilde{V} + \tilde{U} = \left[\underline{v}(r) + \underline{u}(r), \bar{v}(r) + \bar{u}(r)\right]$$
$$\tilde{V} - \tilde{U} = \left[\underline{v}(r) - \underline{u}(r), \bar{v}(r) - \bar{u}(r)\right]$$

3. Newt0n's Meth0d For Unc0nstrained Intuiti0nistic Fuzzy Optimizati0n

Now we consider an unconstrained intuitionistic fuzzy optimization problem $\min_{x\in X}\tilde{G}(x)$ where $\tilde{G}: X \to F(\Re)$ is a fuzzy valued function defined on $X \subseteq \Re^n$. The Newton method is an iterative approach for determining the root of a function f(x). Following an initial estimate to the root x_0, better guesses are provided. The Newton approach for single variable optimization[12] is used to find the root. the Newton method can be adapted to solve unconstrained Intuitionistic Fuzzy Optimization problems, taking into account the unique characteristics of Intuitionistic Fuzzy numbers and their associated membership functions.

$$x_{k+1} = x_k - \left[\frac{f'(x_k)}{f''(x_k)}\right]\ until\ |x_{k+1} - x_k| < \in$$

Newton method for two variable optimization[5] for obtaining the root,

$$x_{k+1} = x_k - \left[\frac{f'(x_k)}{f''(x_k)}\right]\ until\ |x_{k+1} - x_k| < \in$$
$$y_{k+1} = y_k - \left[\frac{f'(y_k)}{f''(y_k)}\right]\ until\ |x_{k+1} - x_k| < \in$$

Newton's approach for determining a function's critical points requires the existence of its first and second derivatives. Newton's approach to finding critical points of a function involves analyzing the behavior of its first and second derivatives. Critical points are locations where the function's rate of change, represented by the first derivative (also known as the derivative or slope), either changes its sign or becomes zero. To determine these points, we need the first derivative to identify where changes in the function's rate of change occur. Additionally, Newton's method requires the second derivative (the derivative of the first derivative) to assess the concavity of the function at these critical points. If the second derivative is positive, the function is concave up, and if it's negative, the function is concave down. This information helps in classifying critical points as local maxima (concave up and negative first derivative), local minima (concave down and positive first derivative), or points of inflection (concave up and positive or concave down and negative first derivative) If the second-order derivatives at iteration $x_{i+1}$are zero, the technique fails. The Newton method's initial derivatives are employed only to find the functions' roots. The second derivatives of the Newton method determine the maximum and least of a given function.

3.1 INTERVAL NEWTON METHOD

The Interval Newton Method shares similarities with the Bisection Method, as it also begins with a bracketing interval. In each iteration, it generates smaller and smaller intervals, all bounded by the intersection with previous iterations. The formula is as follows

$$x_{k+1} = [m(x_k) - \frac{f'(m(x_k))}{f''(x_k)}] \cap (x_k) \qquad (3.1.1)$$

where x's are the interval m(x) is the midpoint of interval x, and f is unconstrained optimization function.

For an fuzzy unconstrained optimization problem then (3.1.1) takes the form

$$\tilde{X}_{k+1}{}^{I} = [m(\tilde{X}_k)^{I} - \frac{f\prime(m(\tilde{X}_k)^{I}}{f''(\tilde{X}_k)^{I}}] \cap (\tilde{X}_k)^{I} \qquad (3.1.2)$$

$$\tilde{X}_{k+1}{}^{I} = [(m(\tilde{X}_{T1}), m(\tilde{X}_{T2}), \dots, m(\tilde{X}_{Tn})) ; (m(\tilde{X}_{F1}), m(\tilde{X}_{F2}), \dots, m(\tilde{X}_{Fn}))] - [\frac{f'[(m(\tilde{X}_{T1}), m(\tilde{X}_{T2}), \dots, m(\tilde{X}_{Tn})) ; (m(\tilde{X}_{F1}), m(\tilde{X}_{F2}), \dots, m(\tilde{X}_{Fn}))]}{f''([((\tilde{X}_{T1}), (\tilde{X}_{T2}), \dots, (\tilde{X}_{Tn})) ; ((\tilde{X}_{F1}), (\tilde{X}_{F2}), \dots, (\tilde{X}_{Fn}))])}] \cap$$

$\left([((\tilde{X}_{T1}), (\tilde{X}_{T2}), \dots, (\tilde{X}_{Tn})) ; ((\tilde{X}_{F1}), (\tilde{X}_{F2}), \dots, (\tilde{X}_{Fn}))]\right)$ where n denotes the intuitionistic fuzzy number. If n=3, it is denoted as intuitionistic triangular fuzzy number, if n=4, it is denoted as intuitionistic trapezoidal fuzzy number and so on.

$$\tilde{X}_{k+1}{}^{I} = [(\left(\frac{\tilde{X}_{T1L} + \tilde{X}_{T1U}}{2}\right), \left(\frac{\tilde{X}_{T2L} + \tilde{X}_{T2U}}{2}\right), \dots, \left(\frac{\tilde{X}_{TnL} + \tilde{X}_{TnU}}{2}\right)) ; (\left(\frac{\tilde{X}_{F1L} + \tilde{X}_{F1U}}{2}\right), \left(\frac{\tilde{X}_{F2L} + \tilde{X}_{F2U}}{2}\right), \dots, \left(\frac{\tilde{X}_{FnL} + \tilde{X}_{FnU}}{2}\right))$$

$$-\left[\frac{f'\begin{bmatrix}\left(\left(\frac{\tilde{X}_{T1L}+\tilde{X}_{T1U}}{2}\right),\left(\frac{\tilde{X}_{T2L}+\tilde{X}_{T2U}}{2}\right),\ldots,\left(\frac{\tilde{X}_{TnL}+\tilde{X}_{TnU}}{2}\right)\right);\\ \left(\left(\frac{\tilde{X}_{F1L}+\tilde{X}_{F1U}}{2}\right),\left(\frac{\tilde{X}_{F2L}+\tilde{X}_{F2U}}{2}\right),\ldots,\left(\frac{\tilde{X}_{FnL}+\tilde{X}_{FnU}}{2}\right)\right)\end{bmatrix}}{f''\left(\begin{bmatrix}\left(\left(\tilde{X}_{T1L},\tilde{X}_{T1U}\right),\left(\tilde{X}_{T2L},\tilde{X}_{T2U}\right),\ldots,\left(\tilde{X}_{TnL},\tilde{X}_{TnU}\right)\right);\\ \left(\left(\tilde{X}_{F1L},\tilde{X}_{F1U}\right),\left(\tilde{X}_{F2L},\tilde{X}_{F2U}\right),\ldots,\left(\tilde{X}_{FnL},\tilde{X}_{FnU}\right)\right)\end{bmatrix}\right)}\right]$$

$$\cap\left(\begin{bmatrix}\left(\left(\tilde{X}_{T1}\right),\left(\tilde{X}_{T2}\right),\ldots,\left(\tilde{X}_{Tn}\right)\right);\\ \left(\left(\tilde{X}_{F1}\right),\left(\tilde{X}_{F2}\right),\ldots,\left(\tilde{X}_{Fn}\right)\right)\end{bmatrix}\right)$$

For Bivariate, the formula

$$X_{k+1} = N_k\big((X_k, Y_k)\big) \cap (X_K)$$

where $N_k\big((X_k, Y_k)\big) = [m(X_k) - \frac{f'\left(m(X_k),m(Y_k)\right)}{f''((X_k,Y_k)}]$

$$X_{k+1} = [m(X_k) - \frac{f'\left(m(X_k),m(Y_k)\right)}{f''(X_k,Y_k)}] \cap (X_k)\ , f''(X_k, Y_k) \neq 0 \qquad (3.1.3)$$

and

$$Y_{i+1} = N_k\big((X_k, Y_k)\big) \cap (Y_K)$$

where $N_k\big((X_k, Y_k)\big) = [m(Y_k) - \frac{f'\left(m(X_k),m(Y_k)\right)}{f''((X_k,Y_k)}]$

$$Y_{k+1} = [m(, Y_k) - \frac{f'\left(m(X_k),m(Y_k)\right)}{f''((X_k,Y_k)}] \cap (Y_k)\ , f''(X_k, Y_k) \neq 0 \qquad (3.1.4)$$

where x's are the interval m(x) is the midpoint of interval x, and f is unconstrained optimization function.

$$\tilde{X}_{k+1}{}^{I} = \left(\begin{bmatrix}\left(m\left(\tilde{X}_{T1}\right),m\left(\tilde{X}_{T2}\right),\ldots,m\left(\tilde{X}_{Tn}\right)\right);\\ \left(m\left(\tilde{X}_{F1}\right),m\left(\tilde{X}_{F2}\right),\ldots,m\left(\tilde{X}_{Fn}\right)\right)\end{bmatrix} - \left[\frac{f'\begin{bmatrix}\left(m\left(\tilde{X}_{T1},\tilde{Y}_{T1}\right),m\left(\tilde{X}_{T2},\tilde{Y}_{T2}\right),\ldots,m\left(\tilde{X}_{Tn},\tilde{Y}_{Tn}\right)\right);\\ \left(m\left(\tilde{X}_{F1},\tilde{Y}_{T1}\right),m\left(\tilde{X}_{F2},\tilde{Y}_{T1}\right),\ldots,m\left(\tilde{X}_{Fn},\tilde{Y}_{T1}\right)\right)\end{bmatrix}}{f''\left(\begin{bmatrix}\left(\left(\tilde{X}_{T1},\tilde{Y}_{T1}\right),\left(\tilde{X}_{T2},\tilde{Y}_{T2}\right),\ldots,\left(\tilde{X}_{Tn},\tilde{Y}_{Tn}\right)\right);\\ \left(\left(\tilde{X}_{F1},\tilde{Y}_{F1}\right),\left(\tilde{X}_{F2},\tilde{Y}_{F2}\right),\ldots,\left(\tilde{X}_{Fn},\tilde{Y}_{Fn}\right)\right)\end{bmatrix}\right)}\right]\right)$$

$$\cap\begin{bmatrix}\left(\left(\tilde{X}_{T1L},\tilde{X}_{T1U}\right),\left(\tilde{X}_{T2L},\tilde{X}_{T2U}\right),\ldots,\left(\tilde{X}_{TnL},\tilde{X}_{TnU}\right)\right);\\ \left(\left(\tilde{X}_{F1L},\tilde{X}_{F1U}\right),\left(\tilde{X}_{F2L},\tilde{X}_{F2U}\right),\ldots,\left(\tilde{X}_{FnL},\tilde{X}_{FnU}\right)\right)\end{bmatrix}$$

$$\tilde{X}_{k+1}{}^{I} = \left[\begin{bmatrix} \begin{pmatrix} \left(\dfrac{\tilde{X}_{T1L}+\tilde{X}_{T1U}}{2}\right), \\ \left(\dfrac{\tilde{X}_{T2L}+\tilde{X}_{T2U}}{2}\right), \\ \dots, \\ \left(\dfrac{\tilde{X}_{TnL}+\tilde{X}_{TnU}}{2}\right) \end{pmatrix} ; \\ \begin{pmatrix} \left(\dfrac{\tilde{X}_{F1L}+\tilde{X}_{F1U}}{2}\right), \\ \left(\dfrac{\tilde{X}_{F2L}+\tilde{X}_{F2U}}{2}\right), \\ \dots, \\ \left(\dfrac{\tilde{X}_{FnL}+\tilde{X}_{FnU}}{2}\right) \end{pmatrix} \end{bmatrix} - \frac{f'\,{}'\begin{bmatrix} \begin{pmatrix} \left(\left(\frac{\tilde{X}_{T1L}+\tilde{X}_{T1U}}{2}\right),\left(\frac{\tilde{Y}_{T1L}+\tilde{Y}_{T1U}}{2}\right)\right), \\ \left(\left(\frac{\tilde{X}_{T2L}+\tilde{X}_{T2U}}{2}\right),\left(\frac{\tilde{Y}_{T2L}+\tilde{Y}_{T2U}}{2}\right)\right),\dots \\ \left(\left(\frac{\tilde{X}_{TnL}+\tilde{X}_{TnU}}{2}\right),\left(\frac{\tilde{Y}_{TnL}+\tilde{Y}_{TnU}}{2}\right)\right); \end{pmatrix}, \\ \begin{pmatrix} \left(\left(\frac{\tilde{X}_{F1L}+\tilde{X}_{F1U}}{2}\right),\left(\frac{\tilde{Y}_{F1L}+\tilde{Y}_{F1U}}{2}\right)\right), \\ \left(\left(\frac{\tilde{Y}_{FnL}+\tilde{Y}_{FnU}}{2}\right),\left(\frac{\tilde{X}_{FnL}+\tilde{Y}_{FnU}}{2}\right)\right)\dots. \\ \left(\left(\frac{\tilde{Y}_{FnL}+\tilde{Y}_{FnU}}{2}\right),\left(\frac{\tilde{X}_{FnL}+\tilde{Y}_{FnU}}{2}\right)\right) \end{pmatrix} \end{bmatrix}}{f''\left(\left(\begin{bmatrix} ((\tilde{X}_{T1},\tilde{Y}_{T1}), \\ (\tilde{X}_{T2},\tilde{Y}_{T2}), \\ \dots, \\ (\tilde{X}_{Tn},\tilde{Y}_{Tn})); \\ ((\tilde{X}_{F1},\tilde{Y}_{F1}), \\ (\tilde{X}_{F2},\tilde{Y}_{F2}), \\ \dots, \\ (\tilde{X}_{Fn},\tilde{Y}_{Fn})) \end{bmatrix}\right)\right)} \right]$$

$$\cap\left(\begin{bmatrix} ((\tilde{X}_{T1L},\tilde{X}_{T1U}),(\tilde{X}_{T2L},\tilde{X}_{T2U}),\dots,(\tilde{X}_{TnL},\tilde{X}_{TnU})); \\ ((\tilde{X}_{F1L},\tilde{X}_{F1U}),(\tilde{X}_{F2L},\tilde{X}_{F2U}),\dots,(\tilde{X}_{FnL},\tilde{X}_{FnU})) \end{bmatrix}\right)$$

$$\tilde{Y}_{k+1}{}^{I} = \left(\begin{bmatrix} (m(\tilde{Y}_{T1}),m(\tilde{Y}_{T2}),\dots,m(\tilde{Y}_{Tn})); \\ (m(\tilde{Y}_{F1}),m(\tilde{Y}_{F2}),\dots,m(\tilde{Y}_{Fn})) \end{bmatrix}\right.$$

$$\left. - \left[\frac{f'\begin{bmatrix} (m(\tilde{X}_{T1},\tilde{Y}_{T1}),m(\tilde{X}_{T2},\tilde{Y}_{T2}),\dots,m(\tilde{X}_{Tn},\tilde{Y}_{Tn})); \\ (m(\tilde{X}_{F1},\tilde{Y}_{T1}),m(\tilde{X}_{F2},\tilde{Y}_{T1}),\dots,m(\tilde{X}_{Fn},\tilde{Y}_{T1})) \end{bmatrix}}{f''\left(\begin{bmatrix} ((\tilde{X}_{T1},\tilde{Y}_{T1}),(\tilde{X}_{T2},\tilde{Y}_{T2}),\dots,(\tilde{X}_{Tn},\tilde{Y}_{Tn})); \\ ((\tilde{X}_{F1},\tilde{Y}_{F1}),(\tilde{X}_{F2},\tilde{Y}_{F2}),\dots,(\tilde{X}_{Fn},\tilde{Y}_{Fn})) \end{bmatrix}\right)}\right]\right)$$

$$\cap\begin{bmatrix} ((\tilde{Y}_{T1L},\tilde{Y}_{T1U}),(\tilde{Y}_{T2L},\tilde{Y}_{T2U}),\dots,(\tilde{Y}_{TnL},\tilde{Y}_{TnU})); \\ ((\tilde{Y}_{F1L},\tilde{Y}_{F1U}),(\tilde{Y}_{F2L},\tilde{Y}_{F2U}),\dots,(\tilde{Y}_{FnL},\tilde{Y}_{FnU})) \end{bmatrix}$$

$$\tilde{X}_{k+1}{}^{I} = \left[\begin{bmatrix} \left(\begin{array}{c} \left(\dfrac{\tilde{Y}_{T1L} + \tilde{Y}_{T1U}}{2}\right), \\ \left(\dfrac{\tilde{Y}_{T2L} + \tilde{Y}_{T2U}}{2}\right), \\ \dots, \\ \left(\dfrac{\tilde{Y}_{TnL} + \tilde{Y}_{TnU}}{2}\right) \end{array} \right); \\ \left(\begin{array}{c} \left(\dfrac{\tilde{Y}_{F1L} + \tilde{Y}_{F1U}}{2}\right), \\ \left(\dfrac{\tilde{Y}_{F2L} + \tilde{Y}_{F2U}}{2}\right), \\ \dots, \\ \left(\dfrac{\tilde{Y}_{FnL} + \tilde{Y}_{FnU}}{2}\right) \end{array} \right) \end{bmatrix} - \frac{f' \begin{bmatrix} \left(\begin{array}{c} \left(\left(\frac{\tilde{X}_{T1L}+\tilde{X}_{T1U}}{2}\right),\left(\frac{\tilde{Y}_{T1L}+\tilde{Y}_{T1U}}{2}\right)\right), \\ \left(\left(\frac{\tilde{X}_{T2L}+\tilde{X}_{T2U}}{2}\right),\left(\frac{\tilde{Y}_{T2L}+\tilde{Y}_{T2U}}{2}\right)\right),\dots \\ \left(\left(\frac{\tilde{X}_{TnL}+\tilde{X}_{TnU}}{2}\right),\left(\frac{\tilde{Y}_{TnL}+\tilde{Y}_{TnU}}{2}\right)\right); \end{array} \right), \\ \left(\begin{array}{c} \left(\left(\frac{\tilde{X}_{F1L}+\tilde{X}_{F1U}}{2}\right),\left(\frac{\tilde{Y}_{F1L}+\tilde{Y}_{F1U}}{2}\right)\right), \\ \left(\left(\frac{\tilde{Y}_{FnL}+\tilde{Y}_{FnU}}{2}\right),\left(\frac{\tilde{X}_{FnL}+\tilde{Y}_{FnU}}{2}\right)\right)\dots. \\ \left(\left(\frac{\tilde{Y}_{FnL}+\tilde{Y}_{FnU}}{2}\right),\left(\frac{\tilde{X}_{FnL}+\tilde{Y}_{FnU}}{2}\right)\right) \end{array} \right) \end{bmatrix}}{f'' \left(\left(\begin{bmatrix} ((\tilde{X}_{T1}, \tilde{Y}_{T1}), \\ (\tilde{X}_{T2}, \tilde{Y}_{T2}), \\ \dots, \\ (\tilde{X}_{Tn}, \tilde{Y}_{Tn})); \\ ((\tilde{X}_{F1}, \tilde{Y}_{F1}), \\ (\tilde{X}_{F2}, \tilde{Y}_{F2}), \\ \dots, \\ (\tilde{X}_{Fn}, \tilde{Y}_{Fn})) \end{bmatrix} \right) \right)} \right]$$

$$\cap \left(\begin{bmatrix} ((\tilde{Y}_{T1L}, \tilde{Y}_{T1U}), (\tilde{Y}_{T2L}, \tilde{Y}_{T2U}), \dots, (\tilde{Y}_{TnL}, \tilde{Y}_{TnU})); \\ ((\tilde{Y}_{F1L}, \tilde{Y}_{F1U}), (\tilde{Y}_{F2L}, \tilde{Y}_{F2U}), \dots, (\tilde{Y}_{FnL}, \tilde{Y}_{FnU})) \end{bmatrix} \right)$$

$$f'' \left(\left(\begin{bmatrix} ((\tilde{X}_{T1}, \tilde{Y}_{T1}), \\ (\tilde{X}_{T2}, \tilde{Y}_{T2}), \\ \dots, \\ (\tilde{X}_{Tn}, \tilde{Y}_{Tn})); \\ ((\tilde{X}_{F1}, \tilde{Y}_{F1}), \\ (\tilde{X}_{F2}, \tilde{Y}_{F2}), \\ \dots, \\ (\tilde{X}_{Fn}, \tilde{Y}_{Fn})) \end{bmatrix} \right) \right) \neq 0^{I}$$

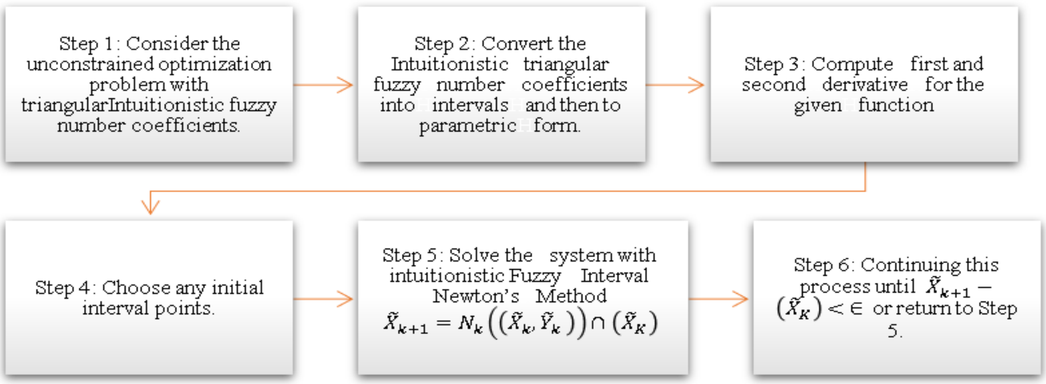
For intuitionistic fuzzy unconstrained optimization problem then (3.1.3) and (3.1.4) takes the form

$$\tilde{X}_{k+1}{}^{I} = N_k{}^{I} \left((\tilde{X}_k, \tilde{Y}_k) \right)^{I} \cap \left(\tilde{X}_K \right)^{I} \qquad (3.1.5)$$

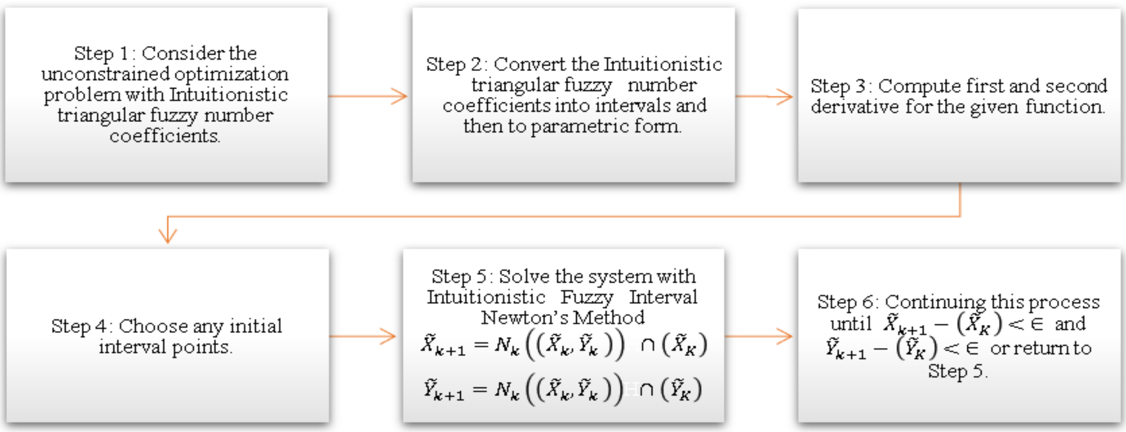
and $\tilde{Y}_{k+1}{}^{I} = N_k{}^{I} \left((\tilde{X}_k, \tilde{Y}_k) \right)^{I} \cap \left(\tilde{Y}_K \right)^{I}$ (3.1.6)

3.2 ALGORITHMS

3.2.1 Algorithm For Single Variable Fuzzy Intuitionistic Interval Newton's Method



3.2.2 Algorithm For Bivariate Intuitionistic Fuzzy Interval Newton's Method :



4. Numerical Illustrations

Example 4.1:

Let us consider the following Unconstrained fuzzy optimization Problem

$$\left(f(\tilde{X})^I\right) = (\tilde{X}^4)^I - (48\tilde{X})^I + (12)^I$$

$f(\tilde{X}) = ((-1,1,3); (-1.1,1,2.9))X^4 - ((47,48,49); (46.9,48,49.1))X + ((10,12,14); (10.5,12,13.5)$

where $X \in \mathfrak{R}$. By employing our proposed arithmetic operations, we first transform all Intuitionistic Triangular Fuzzy numbers into their location index and fuzziness index functions. then the parametric form for unconstrained optimization problem is written as

$f(\tilde{X}) = ((-1,1,3); (-1.1,1,2.9))X^3 - ((47,48,49); (46.9,48,49.1))X + ((10,12,14); (10.5,12,13.5)$

$f'(\tilde{X}) = 4((-1,1,3);(-1.1,1,2.9))X^3 - ((47,48,49);(46.9,48,49.1))$

$f'(\tilde{X})$=$((-1,1,3);(-1.1,1,2.9))X^3 - ((11.75,12,12.25);(11.725,12,12.2750)$

$f''(\tilde{X})$ =$3((-1,1,3);(-1.1,1,2.9))X^2$

The Interval analysis of Newton method for unconstrained fuzzy optimization is $(\tilde{X}_{i+1})^I = N(\tilde{X})^I \cap (\tilde{X}_i)^I$

Here $N(\tilde{X})^I = [m(\tilde{X})^I - \frac{f'(m(\tilde{X})^I}{f''(\tilde{X})^I}]$

Let us take the initial interval $\tilde{X}_0 = [((1,2,3);(1.1,2,2.9)),((3,4,5);(3.1,4,4.9))]$ in Intuitionistic triangular fuzzy number.

$m(\tilde{X}_0)^I = [(2,3,4);(2.1,3,2.9)]$

$$N(\tilde{X})^I = m(\tilde{X}_0)^I - \left[\frac{(((-1,1,3);(-1.1,1,2.9)) * [(2,3,4);(2.1,3,3.9)]^3 \\ -((11.75,12,12.25);(11.725,12,12.2750)}{3[((1,2,3);(1.1,2,2.9)),((3,4,5);(3.1,4,4.9))]^2}\right]$$

$$= [(2,3,4);(2.1,3,2.9)] - \left[\frac{(((-1,1,3);(-1.1,1,2.9)) * [(2,3,4);(2.1,3,3.9)]^3 \\ -((11.75,12,12.25);(11.725,12,12.2750)}{3[((2,4,6);(2.2,4,5.8),(12,16,20);(12.4,16,19.6)]}\right]$$

$$=\begin{bmatrix} ((0.5208,1.25,1.9791);(0.59791,1.25,1.9020)), \\ (0.13021,0.31250.49479);(0.14947,0.3125,0.4755)) \end{bmatrix}$$

$$(\tilde{X}_1)^I = [((1,2,3);(1.1,2,2.9)),((3,4,5);(3.1,4,4.9))] \cap \begin{bmatrix} ((0.5208,1.25,1.9791);(0.59791,1.25,1.9020)), \\ (0.13021,0.31250.49479);(0.14947,0.3125,0.4755)) \end{bmatrix}$$

$$(\tilde{X}_1)=\begin{bmatrix} ((1,2,3);(1.1,2,2.9)), \\ ((3,4,5);(3.1,4,4.9)) \end{bmatrix} \cap \begin{bmatrix} (1.375,1.75,2.125);(1.4,1.75,2.1), \\ (1.4687,2.6875,3.9062);(1.55,2.6875,3.825) \end{bmatrix}$$

$$(\tilde{X}_1) = \begin{bmatrix} (1,2,3);(1.1,2,2.9), \\ (1.4687,2.6875,3.9062);(1.55,2.6875,3.825) \end{bmatrix}$$

$$(\tilde{X}_2) = \begin{bmatrix} (1.6693,2.2708,2.8724);(1.7184,2.2708,2.8232), \\ (1.47524,2.0333,3.1315);(1.5429,2.3033,3.0638) \end{bmatrix}$$

Thus, f(x) is minimum at $x = 2.28675^i$, instead the elementary calculus technique produces minima at $x = 2.28675^i$.

Example 2:

Consider x and y as distinct diagnostic markers or criteria for medical diagnosis. These signals could be anything from specific lab readings to clinical complaints or imaging results. The equation then reflects the interaction of various diagnostic signs in evaluating a patient's status. In a medical context, x and y may indicate various clinical aspects or biomarkers associated with a specific disease or condition. For example, x might indicate blood pressure data, and y could represent blood glucose levels in a diabetic patient. The equation $x^3 - 3xy + y^3$ could represent a diagnostic situation in which the values of certain indicators interact in a given manner, resulting in a particular diagnosis or clinical consequence. The equation may be analyzed to determine how changes in one diagnostic marker affect the interpretation of another, as well as how these interactions influence the diagnostic process.

Let us consider the following Unconstrained fuzzy optimization Problem with intuitionistic triangular$(0.5,1,1.5);(0.4,1,1.6)x^3 - (2,3,4);(2.1,3,3.9)xy + (0.5,1,1.5);(0.4,1,1.6)y^3$, Initial interval intuitionistic value for

$$X_0 = [(0.5,1,1.5);(0.4,1,1.6),(1,2,3);(0.9,2,3.1)]\ and$$

$$Y_0 = [(0.5,1,1.5);(0.4,1,1.6),(1,2,3);(0.9,2,3.1)]$$

m$(X_k) = [(0.75,1.5,2.25);(0.65,1.5,2.35)]$

m$(Y_k) = [(0.75,1.5,2.25);(0.65,1.5,2.35)]$

$f'(X_k) = ((0.375,0.75,1.125);(0.325,0.75,1.175))$ and

$f'(Y_k) = ((0.375,0.75,1.125);(0.325,0.75,1.175))$

$f''(X_k) = [((1,2,3);(0.8,2,3.2)),((2,4,6);(1.8,4,6.2))]$ and

$f''(Y_k) = [((1,2,3);(0.8,2,3.2)),((2,4,6);(1.8,4,6.2))]$

$$X_1 = \left[\frac{[(0.75,1.5,2.25);(0.65,1.5,2.35)] - \big((0.375,0.75,1.125);(0.325,0.75,1.175)\big)}{\big[\big((1,2,3);(0.8,2,3.2)\big),\big((2,4,6);(1.8,4,6.2)\big)\big]}\right] \cap$$

$$[(0.5,1,1.5);(0.4,1,1.6),(1,2,3);(0.9,2,3.1)]$$

$$X_1 = \begin{bmatrix} \big((0.5625,1.125,1.6875);(0.4875,1.125,1.7625)\big), \\ ((0.6562,1.3125,1.9687);(0.5688,1.3125,2.0563) \end{bmatrix} \cap$$

$$[(0.5,1,1.5);(0.4,1,1.6),(1,2,3);(0.9,2,3.1)]$$

$$X_1 = \begin{bmatrix} \big((0.5625,1.125,1.6875);(0.4875,1.125,1.7625)\big), \\ ((0.6562,1.3125,1.9687);(0.5688,1.3125,2.0563) \end{bmatrix}$$

And

$$Y_1 = \begin{bmatrix} \big((0.5625,1.125,1.6875);(0.4875,1.125,1.7625)\big), \\ ((0.6562,1.3125,1.9687);(0.5688,1.3125,2.0563) \end{bmatrix}$$

$$X_2 = \begin{bmatrix} \big((0.5625,1.125,1.6875);(0.4875,1.125,1.7625)\big), \\ \big((0.5586,1.1172,1.6758);(0.4841,1.1172,1.7503)\big) \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} \big((0.5625,1.125,1.6875);(0.4875,1.125,1.7625)\big), \\ ((0.5586,1.1172,1.6758);(0.4841,1.1172,1.7503)) \end{bmatrix}$$

In a hypothetical scenario where x denotes blood pressure and y represents blood glucose levels, a negative coefficient like -3 implies that an increase in blood pressure is associated with a decrease in blood glucose levels. This relationship could potentially indicate the presence of a specific medical condition or risk factor that influences both blood pressure and glucose levels in this manner. However, to confirm such a connection, further research and analysis involving larger datasets and expert input would be necessary.

5. Conclusion

The proposed paper introduces an innovative approach to tackle interval-based Intuitionistic Fuzzy optimization problems. By employing triangular interval-valued fuzzy numbers and the interval Newton's method, it aim to provide a reliable solution for these complex issues. The effectiveness of this method is demonstrated through the presentation of relevant numerical examples.

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