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Neutrosophic Pre Generalized Pre Regular Star Weakly Closed Sets

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Abstract

The objective of this paper is to emphasis the generalization of pre generalized pre regular weakly closed sets in Neutrosophic environment namely Neutrosophic Pre generalized Pre Regular Star Weakly Closed Sets. Its properties and characterizations are defined and its relationships with other Neutrosophic Sets are studied with suitable examples.

Keywords: N-Pgpr*wC(X), N-R*gαC(X), N-Pgpr*gwO(X), N-R*gαO(X).

1. Introduction:

. In Recent Times Many Researchers undergoing their research in the area.of g clossd sets In 1965 L.Zadeh [10] introduced the concept of fuzzy sets which deals with membership of a set. K.Atanassova [1] introduced intuitionistic fuzzy sets with membership and nonmembership function.

F. Smarandache [2] developed his new concept namely Neutrosophic set that studies membership, nonmembership and indeterminacy. Later on A.A.Salama and S.A.Alblow [6] introduced neutrosophic topological spaces by using the neutrosophic sets. In the year 2015 R.S.Wali and Vivekananda Dembre [9] introduced and studied Pre generalized Pre Regular Weekly closed and open sets respectively. In this paper we apply the concept and properties of Neutrosophic sets to develop a new class of set called Neutrosophic Pre-generalized pre regular star weakly closed set in Neutrosophic topological spaces.

In this case the pair (X, τ) is a neutrosophic topological space and any neutrosophic set in X is known as a neutrosopic open set (N-OS) in X. A neutrosophic set S is a neutrosophic closed set (N-CS) if and only if C(S) is a neutrosophic open set in X. Here the empty set (O_N) and the whole set (I_N) may be defined as follows

We go through some basic definitions in this section.

Definition 2.1: [2] Let X be a non-empty fixed set. A neutrosophic set (N-S) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where

$\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A neutrosophic set $A = \{\langle x, \mu A(x), \sigma A(x), vA(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu A(x), \sigma A(x), vA(x)\rangle$ in $]^{-}0, 1^{+}[$ on X.

Definition 2.2: [6] Let $A = \{\langle x, \mu A(x), \sigma A(x), vA(x)\rangle : x \in X\}$ be a NS on X, then the complement C(A) may be defined as

1. $C(A) = \{\langle x, 1-\mu A(x), 1-vA(x)\rangle : x \in X\}$
2. $C(A) = \{\langle x, \mu A(x), \sigma A(x), vA(x)\rangle : x \in X\}$
3. $C(A) = \{\langle x, vA(x), 1-\sigma A(x), \mu A(x)\rangle : x \in X\}$ Note that for any two neutrosophic sets A and B,
4. $C(A \cup B) = C(A) \cap C(B)$
5. $C(A \cap b) = C(A) \cup C(B)$

Definition 2.3: [6] For any two neutrosophic sets $A = \{\langle x, \mu A(x), \sigma A(x), vA(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu B(x), \sigma B(x), vB(x)\rangle : x \in X\}$ we may have.

1. $A \subseteq B \Leftrightarrow A\ \mu A(x) \le \mu B(x), \sigma A(x) \le \sigma B(x)$ and $vA(x)\rangle \ge vB(x)\ \forall\ x \in X$
2. $A \subseteq B \Leftrightarrow A\ \mu A(x) \le \mu B(x), \sigma A(x) \ge \sigma B(x)$ and $vA(x)\rangle \ge vB(x)\ \forall\ x \in X$
3. $A \cap B = \langle x, \mu A(x) \wedge \mu B(x), \sigma A(x) \wedge \sigma B(x), vA(x)\rangle \vee vB(x)\rangle$
4. $A \cap B = \langle x, \mu A(x) \wedge \mu B(x), \sigma A(x) \vee \sigma B(x), vA(x)\rangle \vee vB(x)\rangle$
5. $A \cup B = \langle x, \mu A(x) \vee \mu B(x), \sigma A(x) \vee \sigma B(x), vA(x)\rangle \wedge vB(x)\rangle$
6. $A \cup B = \langle x, \mu A(x) \vee \mu B(x), \sigma A(x) \wedge \sigma B(x), vA(x)\rangle \wedge vB(x)\rangle$

Definition 2.4: [6] A neutrosophic topology on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms: (NT1) (NT1) $0_N, 1_N \in \tau$

(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(NT3) $\cup G_1 \in \tau\ \forall\ \{G_i : i \in j\} \subseteq \tau$

In this case the pair (X, τ) is a neutrosophic topological space and any neutrosophic set in τ is known as a neutrosopic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement C(A) is a neutrosophic open set in X. Here the empty set (O_S) and the whole set (I_N) may be defined as follows:

(01) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(02) $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

(03) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

(04) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

(11) $1_N = \{\langle x, 1, 0, 0\rangle: x \in X\}$

(12) $1_N = \{\langle x, 1, 0, 1\rangle: x \in X\}$

(13) $1_N = \{\langle x, 1, 1, 0\rangle: x \in X\}$

(14) $1_N = \{\langle x, 1, 1, 1\rangle: x \in X\}$

Definition 2.5: [6] Let (X, τ) be a N-TS and $A = \{\langle x, \mu A(x), \sigma A(x), \nu A(x)\rangle: x \in X\}$ be a N-S in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

NInt(A) = ∪ {G : G is an NOS in X and G ⊆ A}

NInt(A) = ∩ {K : K is an NOS in X and A ⊆ K}

Note that for any NS A, NCl(C(A)) = C(NInt(A)) and NInt(C(A)) = C(NCl(A)).

Definition - 2.6: [3] N-S $S = \{\langle x, \mu A(x), \sigma A(x), \nu A(x)\rangle: x \in X$ in a N-TS

The N-S $S = \{\langle x, \mu A(x), \sigma A(x), \nu A(x)\rangle: x \in X\}$. is said to be Neutrosophic

- (regular - (N-RCS) ,semi- (N-SCS), pre- (N-PCS), (N-αCS)) if
- (S = N-Cl(N-INT(S)), N-INT(N-Cl(S)) ⊆ S , N-Cl(N-INT(S)) ⊆ S, N-Cl(N-INT(N-Cl(S)))

⊆ S.)

- The complement of the above closed sets are their respective open sets

Definition 2.7: [8] The N-S A is said to be Neutrosophic generalized pre closed set (N-gpCS) if N-PCl(A) ⊆ U whenever A ⊆ U and U is NOS in (X, τ). C(N-gpCS) in (X,τ) is called Neutrosophic generalized preopen set (N-gpOS shortly).

Definition 2.8: The N-S A is said to be Neutrosophic generalized pre Regular closed set (N-gprCS shortly) if N-PCl(A) ⊆ U whenever A ⊆ U and U is N-ROS in (X, τ). .

Definition 2.9: [3] The N-S A is said to be Neutrosophic Regular generalized closed set (N-RgCS shortly) if N-Cl(A) ⊆ U whenever A ⊆ U and U is N-ROS in (X,τ). .

Definition 2.10: The N-S A is said to be Neutrosophic generalized pre Regular weakly closed set (N-gprwCS shortly) if N-PCl(A) ⊆ U whenever A ⊆ U and U is N-RSOS in (X,τ). .

Definition 2.11: The N-S A is said to be N-R*α-closed set (N-R*αCS shortly) if U is N-RCS such that A ⊆ U ⊆ N=αInt(U).

Definition 2.12: The N-S A is said to be Neutrosophic R*gα-closed set (N-R*gαCS shortly) if N-αCl(A) ⊆ U whenever A ⊆ U and U is N-R*αOS in (X,τ).

The Complement of the above closed sets are their respective open sets

3. Neutrosophic Pre generalized Pre regular star weakly closed sets

Definition 3.1: The N-S S is Nutrosophic pre generalised pre regular star weakly Closed (N-Pgpr*wCS) if N-PCl(S) ⊆ M whenever S ⊆ M and M is N-R*gαOS in X. C(N-Pgpr*wCS) is

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Neutrosophic pre generalized pre regular star weakly open- sets (N-Pgpr*wOS). The collection of all N-Pgpr*wCS of N-TS (X,τ) is denoted by N-PGPR*WC-X. The N-TS (X,τ) is denoted by N-TS. X or X

Example 3.2

Let X = { a, b } and τ = { 0_N, S, M, 1_N }.

where M = ⟨(0.2, 0.2, 0.7), (0.3, 0.1, 0.7)⟩

V = ⟨(0.8, 0.2, 0.2), (0.7, 0.2, 0.2)⟩

Then X is a N-TS.

Here the N-S S= ⟨(0.1, 0.2, 0.7), (0.2, 0.2, 0.8)⟩ is a N-Pgpr*wCS in X. Since S ⊆ M and V is N- R*gαOS since N-PCl(S) = S ⊆ M.

Proposition- 3.3

In (X,τ) let S be a N-S satisfying the following properties Every

1. N-pCS is N-Pgpr*wCS

2. N-CS is N-Pgpr*wCS

3. N-RCS is N-Pgpr*wCS

4. N-gCS is N-Pgpr*wCS

5. N-alphaCS is N-Pgpr*wCS

6. N-WCS is N-Pgpr*wCS

7. N-Pgpr*wCS is N-gpCS

8. N-Pgpr*wCS is N-gprCS

Proof.

1. Let M be a N-R*gαCS in X As **S is NpCS**, then N-Cl(N-int (S)) ⊆ S. so N-PCl (S) = S ∪ N-Cl(N-int (S)) ⊆ S∪ S = S ⊆ M Hence S is N-Pgpr*wCS in X.
2. Let S ⊆ M and, M be a N-R*gαCS in X As **S is N-CS** in X, N-Cl (S) = S. Implies N-PCl (S) ⊆ N-Cl(S) = S ⊆ M by hypothesis. S is a NPgpr*wCS in X.
3. Let S ⊆ M and M be a N-R*gαOS As every **N-RCS** is N-CS then N-Cl(S) = S. By hypothesis N-PCl (S) ⊆ N-Cl (S) ⊆ M and N-PCl(S) ⊆ M Thus S is N-Pgpr*wCS
4. Let M be a N-R*gαOS such that S ⊆ M. Since every **N-gCS** is a N-CS. We have N-Cl(S) = S. By hypothesis, N-PCl(S) ⊆ N-CL(S) ⊆ M Hence N-PCl(S) ⊆ M. Thus S is N-Pgpr*wCS.
5. Let S ⊆ M and M be a N-R*gαOS in X As S is N-αCS N-Cl (N-int (N-Cl (S)) ⊆ S. Also S ⊆ N-Cl (S) implies N-Cl (N-int(S)) ⊆ N-Cl(N-Int (N-Cl(S)) ⊆ S. Implies N-PCl(S) = S∪N-Cl(N-int(S)) ⊆ S∪S = S⊆ M. Therefore S is N-Pgpr*wCS in X

6. Let S ⊆ M,and M be a N-R*gαOS in X.As **every N-wCS** is N-CS. Implies N-CL (S) = S. by hypothesis, N-PCl(S) ⊆ N-Cl(S) ⊆ M then N-PCl(S) ⊆ S. since S is N-OS. We have N-Cl (S) = S Therefore S is N-Pgpr*wCS in X

7. Let M **be N-OS** in X such that S ⊆ M. Since S is **N-Pgpr*wCS** in X, and every N-OS is N-R*gαOS in X.We have M is N-R*gαOS in X

Therefore, N-PCl(S) ⊆ M. Hence S is N-gpCS in X

8 Let S be **N-Pgpr*wCS** and M be N-POS such that S ⊆ M. As every N-ROS is N-R*gαOS

M is N-R*gαOS in X

Since S is N-Pgpr*wCS. Then N-PCl(S) ⊆ U. Therefore S is **N-gprCS**.

Remark: 3.4 The converse of subdivision 1 to 5 of theorem 3.3 need not be true can be proved by the following

example

Example 3.5

As X , τ, U, M defined in ex 3.2

N-S S= ⟨(0.8, 0.2, 0.1), (0.8, 0.2, 0.1)⟩ is a N-Pgpr*wCS in X S is not N-CS, N-PCS, N−αCS,N-RCS,N-WCS, N-GCS

Remark: 3.6 The converse of subdivision 6 and 7 of theorem 3.3 proved by the following example

Example 3.7

As X , τ, defined in ex 3.2

M = {(0.5, 0.2, 0.5), (0.2, 02, 0.8)}

Here the N-S S = ⟨ (0.2,0.2, 0.6) (0.2 0.2, 0.8) ⟩

S is N-gpCS

As S ⊆ M, M is N-R*gαOS

Also N-PCl(S) = M^C ⊆ M,. N-Cl(S) = M^C ≠ S.

Implies S is not N-Pgpr*wCS

Example 3.8

From Ex 3.7 S is N-gprCS

The N-S S = ⟨(0.5, 0.2, 0.5), (0.2, 0.2, 0.8)⟩ is N-gprCS

Since N-PCl(S) = S ⊆ M whenever S ⊆ M where M is N-ROS.

But N-α PCl(S) = M^C ⊈ M. The N-S S is not N-Pgpr*wCS.

Theorem 3.9: If S is N-ROS and N-Pgpr*wCS in X then S is N-pCS **Proof:**

Let S be N-ROS & N-Pgpr*wCS as every N-ROS is N-R*gαOS . Since S ⊆ S and S is N-

Communications on Applied Nonlinear Analysis
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Vol 31 No. 3s (2024)

Pgpr*wCS.

We have N-PCl(S) $\subseteq$ S and Also S $\subseteq$ N-PCl(S)

N-PCl(S) = S. Hence S is N-PCS.

Theorem 3.10: If S is N-OS and N-gpCS then S is N-pgr*wCS.

Proof:

Let S be N-OS and N-gpCS.

Let M be N-R*gαOS such that S $\subseteq$ M.

Since S is N-OS and N-gpCS. We have N-PCl(S) $\subseteq$ S $\subseteq$ M. So, N-PCl(S) $\subseteq$ M whenever S $\subseteq$ M.

Therefore S is N-Pgpr*wCS.

Theorem 3.11: The Union of two N-Pgpr*wCS of X is N-Pgpr*wCS.

Proof:

Let C and D be N-Pgpr*wCS in X.By definition

C,D $\subseteq$ M and M be a N-R*gαOS in X where C $\subseteq$ M and D $\subseteq$ M. Then N-PCl(C∪D) = N-PCl(A)∪ N-PCl(S) $\subseteq$ M by hypothesis.

Hence C∪D is also N-Pgpr*wCS in X.

Theorem 3.12: The Intersection of two N-Pgpr*wCS in X is generally not an N-Pgpr*wCS in X.

Theorem 3.13: If S is N-Pgpr*wCS and S $\subseteq$ C $\subseteq$ N-PCl(S). Then C is also Npgpr*wCS in X.

Proof:

Let S be Npgpr*wCS in X.

To prove C is Npgpr*wCS in X.

Let M be an N-R*gαOS in X such that C $\subseteq$ M. Since S is N-Pgpr*wCS and S $\subseteq$ C. We have N-PCl(S) $\subseteq$ M and S $\subseteq$ M.

Now C $\subseteq$ N-PCl(S)∪ N-PCl(C) $\subseteq$ N-PCl(N-PCl(S)) $\subseteq$ N-PCl(S) $\subseteq$ M.

Therefore N-PCl(S) $\subseteq$ M. Hence C is N-Pgpr*wCS in X.

Theorem 3.14: If a subset S is both N-SOS and N-wCS then S is N-Pgpr*wCS in X..

Proof:

Let S be N-S open and N-wCS in X.

:

Let S $\subseteq$ M and M be N-R*gαOS in X. Now S $\subseteq$ S. By hypothesis, N-CI(S) $\subseteq$ S. Therefore N-PCl(S) $\subseteq$ N-Cl(S) $\subseteq$ S $\subseteq$ M. Hence S is N-Pgpr*wCS in X.

Theorem 3.15: If S is both N-ROS and N-RgCS then S is N-Pgpr*wCS

Proof:

Let S $\subseteq$ M where M is N-R*gαOS

Since S is N-ROS and N-RgCS. We have S $\subseteq$ S. So N-Cl(S) $\subseteq$ S then N-PCl(S) $\subseteq$ N-Cl(S).

Therefore N-PCl(S) $\subseteq$ S, whenever S $\subseteq$ M. proves· S is N-Pgpr*wCS

Theorem 3.16: If S is both N-RSOS and N-gprwCS then it is N-Pgpr*wCS.

Proof:

Let S be N-RSOS and N-gprwCS,

M be N-R*gαOS in X such that S $\subseteq$ M By hypothesis, S $\subseteq$ S.

Therefore N-PCl(S) $\subseteq$ S $\subseteq$ M. So N-PCl(S) $\subseteq$ M.

Hence S is a N-Pgpr*wCS.

Remark 3.17: To prove the converse need not be true As X , τ, defined in ex 3.2

Where M = ⟨x, (0.7,0.2, 0.5),(0.8,0.2, 0..3)⟩

U = ⟨x, (0.2,0.2,0.9) (0.3 0.2, 0.9)⟩

A = ⟨x, (0.3,.0.2,0.7) (0.3,0.2, 0.8)⟩

N-S S is N-Pgpr*wCS when M is N-R*gαOS . As N-PCl(A) = M^C $\subseteq$ M whenever S $\subseteq$ M.

Sut since S$\not\subseteq$ N-CL(intS) i.e. S$\not\subseteq$ M^C.

∴ S is not N-RSOS and thus S is not N-gprwCS.

Theorem 3.18: If A is both N-OS and N-gCS then S is N-Pgpr*wCS.

Proof:

Given S is N-OS and N-gCS.

Let M be any N-R*gαOS such that S $\subseteq$ M. Since S $\subseteq$ S and N-OS and S is N-gCl(S).

Hence N-Cl(S) $\subseteq$ S and N-PCl(S) $\subseteq$ N-Cl(S) $\subseteq$ S $\subseteq$ M.

Thus N-PCl(S) $\subseteq$ M whenever S $\subseteq$ M and M is N-R*gαOS in X,. Therefore S is N- Pgpr*wCS.

Theorem 3.19: If S is N-ROS and N-gprCS then it is N-Pgpr*wCS.

Proof:

If A is N-ROS and N-gprCS.

Let M be any N-R*gαOS such that S $\subseteq$ M. Thus N-PCl(S) $\subseteq$ S and N-PCl(S) $\subseteq$ M. Therefore N-PCl(S) $\subseteq$ M whenever S $\subseteq$ M and M is N-R*gαOS in X. Therefore S is N-

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Pgpr*wCS.

Theorem 3.20: If S is both N-RSOS and N-gprwOS then it is N-pgpr*w CLosed.

Proof:

Subset

Let S be N-RS open and N-gprw Closed.

Let M be N-R*gαOS open in X such that S⊆ M

By hypothesis, S ⊆ S. Therefore, N-PCl(S) ⊆ S ⊆ M So, N-PCl(S) ⊆ M. Thus, S is N-Pgpr*wCS.

Theorem 3.21: If S in X such that K ⊆ N-PCl(S)-S then K = **ϕ** .where K is a non-empty N-R*gαOS of N-PCl(S)-S

Proof:

Let S be N-Pgpr*wCS in X. Given K ⊆ **N-**PCl(S)-S

K ⊆ **N-**PCl(S)-S implies K ⊆ N-PCl(S)∩ S

K ⊆ N-PCl(S) (1)

K ⊆ **X-S and S** ⊆ X-K

we have X - K is N-R*gαOS and S is Npgpr*wCS .implies

N-PCl(S) ⊆ X-K.. Therefore, K ⊆ X-N-PCl(S) (2)

From (1) and (2), K ⊆ N-PCl(S)intersection(X-N-PCl(S)) = **ϕ**

Implies K = **ϕ**.

Thus, N-PCl(S)-S does not contain any non empty N-R*gα CS.

Theorem 3.22: Let S be N-Pgpr*wCS in X. Thus S is N-pCS iff N-PCl(S) -S is N-RCS.

Proof:

Suppose S is N-pCS.

Then N-PCl(S) = S. So, N-PCl(S)-S = **ϕ** which is N-RCS. Conversely, Suppose S is N-Pgpr*wCS and N-PCl(S) = S is N-RCS. By the theorem, N-PCl(S) - S implies N-PCl(S) = S.

Therefore, S is N-pCS

4. Neutrosophic pre generalized pre regular star weakly Open sets

We go through some basic definitions in this section.

Definition 4.1: The NS A is said to be Neutrosophic pre generalised pre Regular weakly open set (N-pgpr*wOS shortly) if N-pINT(S) ⊃M whenever S⊃M and M is N-R*gαCS in X. The family of all N'pgpr*wOS of N-TS (X,τ) is denoted by N-PGPR*WO-X.

Example 4.2:

Let X = {a, b} and τ = {0_N, L, M, 1_N}.

where M = ⟨(0.5, 0.3, 0.6), (0.4, 0.4, 0.7)⟩

L = ⟨(0.7, 0.5, 0 .3) , (0.7, 0.5, 0.2)⟩

then X is a N-CS.

Here the N-S S = ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is a N-Pgpr*wOS in X, . Since S ⊃M^C and M^C is a N-R*gαCS , As we have N-pINT(S) = S ⊃M^C.

Theorem 4.3: Every N-OS is N-pgpr*wOS. but the converse may not be true.

Proof:

Let M be N-R*gαCS in X such that S ⊃M. Since S is N-OS, N-pINT(S) = S,

By hypothesis,

. N-pINT(S) = S ∩ N-Int(N-Cl(S)) = S ∩ N-Cl(S) ⊃ S∩S =ScontainsM

Therefore S is N- Pgpr*wOS in X.

Example 4.4

In Example 4.2 the N-S S = ⟨(0.8, 0.9, 0.2), (0.9, 0.6, 0.1)⟩ is an N-Pgpr*wOS in X, but not a N-OS in X.

Theorem 4.5: For any Nuetrosophic Topological Space X,. We have the following

1. Every N-ROS, N-OS, N-WOS, N-POS, N-GOS is a N-Pgpr*wOS But the converse need not true.

2. Every N-gpOS, N-gprOS is N-Pgpr*wOS but the converse need not true in general.

Remark 4.6 converse of theorem 4.5 can be proved by the following example to show it is not true

1. M = ⟨(0.5, 0.3, 0.4) , (0.9, 0.7, 0.8)⟩. Then (X,) is a N-CS.

Here the N-S S = ⟨(0.3, 0.9, 0 .7) , (0.7, 0.5, 0.9)⟩ is a N-Pgpr*wCS in X, . Since S ⊃ M we have N-pINT(S) = 1_N ⊃ 1_N,But since N-INT(N-Cl(S)) = 1_N ≠ S ,

S is not a N-ROS, Similarly S is **not N-OS, N-WOS, N-POS and N-GOS in (X,)**

2. Let X = {a, b} and = {0_N, M, 1_N}

where M = ⟨(0.4, 0.2, 0.3) , (0.8, 0.6, 0.7)⟩. Then X, is a N-CS.

M= ⟨(0.2, 0.1, 0.8) , (0.2, 0.1, 0.8)⟩

Here the N-S S = ⟨(0.5, 0.9, 0.5) , (0.8, 0.9, 0.2)⟩ is a N-Pgpr*wCS in X, . Since S ⊃ 0_N, we have N-pINT(S) = 0_N ⊃ 0_N, so S is not a **N-gpOS**,

N-gprOS in X, .

Theorem 4.7: The intersection of two N-Pgpr*wOSis N-pgpr*wOS.

Proof:

Let C and D Be the N-Pgpr*wOSin X

Let C∩ D⊃M and M Be N-R*gαOS in X where C⊃ M and D⊃ M Then N -*P*int(C∩D)= (C∩D)∩N-pINT(N-Pcl(N-pINT(C∩D)))

(C∩ D) ∩ N-pINT(C∩ D)

= (C∩ D)∩ N-pINT(C))∩ N-pINT(D)⊃ M

By hypothesis

Hence C∩ D is also N-Pgpr*wOS in X.

theorem 4.8: The N-S S of NCS X, is a N-Pgpr*wOSin X, iff M ⊆ N-pINT(S) whenever M is a N-R*gαCS in X, and M ⊆ S

Proof:

Let S be N-Pgpr*wOSin X and M is N-R*gαCS in X, such that S⊃M .

then X-S is N-Pgpr*w-closed in X.

Also X-S ⊆ X-M and X-M is N-R*gαOS in X. Hence N-PCl(X-S) ⊆ X-M

We know that N-PCl(X-S) = X - N-pINT(S)

∴ X-N-pINT(S) ⊆ X-M. So, M ⊆ N-pINT(S).

Conversely, Suppose M ⊆ N-pINT(S) whenever M is N-R*gαCS and S⊃M . To prove S is N-Pgpr*wOS.

Let F be N-R*gαCS of X such that X-S ⊆ F

then X-F ⊆ S. Now X-F is N-R*gαCS contained in S. So X-F ⊆ N-pINT(S) implies X-N-pINT(S) ⊆ F. But N-PCl(X-S) = X-N-pINTS ⊆ F.

therefore X-S is N-Pgpr*wCS. Hence S is N-Pgpr*wOS.

Theorem 4.9: If S ⊆ X is N-pgpr*wCS in X and F = **ϕ** then N-PCl(S)-S is N-Pgpr*wOS.

Proof:

Let S be N-pgpr*wCS and F = **ϕ**. Let S ⊆ N-PCl(A)-A where F is N-R*gαCS. Then By the theorem N-PCl(S)-S does not contain N-R*gαCS in X.

i.e. F = **ϕ**. Therefore F ⊆ N-pINT(N-PCl(S)-S).

N-PCl(S)-S is N-Pgpr*wOS

Theorem 4.10: If S be N-S is N-Pgpr*Wos in X iff M = X whenever M is N-R*gαCS and M^C ⊆

((N-pINT(S)) $S^C)^C$.

Proof:

Suppos S is N-Pgpr*wOS in X. Let M be N-R*gαCS and

$M^C \subseteq$ ((N-pINTS) $S^C)^C M^C \subseteq$ (N-pINT (S))$^C \cap$ S

$M^C \subseteq$ (N-pINTS)^C − S . $M^C \subseteq$ N-PCl (S^C) − S^C

M'^{C} is N-R*gαCS

∴ M^C = ϕ and therefore M = X

Theorem 4.11: Let X be a N-TS. and S be a N-Pgpr*wOS in X. then X = S·

We know that N-PCl(X-S) = X-N-pINT(S)

N-PCl(X-S) + N-Pint(S) = S N-PCl(S)^C + N-pINT(S) = S (N-pINTS)^C + N-pINTS = S

Theorem 4.12: If S and C are Neutrosophic sets of X. If C is N-Pgpr*Wos and N-pINT(S) $\subseteq$ S then $C^C \subseteq$ N-PCl (C^C) $\subseteq S^C$.

Proof:

N-pINT(C) $\subseteq$ S and C $\subseteq$ N-pINT(C)

C $\subseteq$ N-pINT(C) $\subseteq$ S

X – C $\subseteq$ X – N-pINT(C) $\subseteq$ X-S $C^C \subseteq$ N-PCl(X-C) $\subseteq S^C$

$C^C \subseteq$ N-PCl(C^C) $\subseteq S^C$.

: **Theorem 4.13:**

Let S be a N-pgpr*WOS in X and S ⊇ C ⊇ N-pInt(S), then C is N-pgpr*WOS in X

Proof:

Let S be aa N-pgpr*WOS in X and C be a neutrosophic set in X. Let S⊇ C ⊇ N-PInt(S). Then Sc is a N-pgpr*WCS in X and Sc ⊆ Cc ⊆ N-PCl(Sc). Then Cc is a N-pgpr*WCS set in X . Hence C is a N-pgpr*WOS in X.

References:

[1] AtanassoM K.C., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986, 20, 87-96.

[2] Floretin Smarandache, Neutrosophic Set: A Generalization of Intuitional Fuzzy Set, Journal of Defense, 2010, 1, 107-116.

[3] Harshitha, A. and Jayanthi, D., Regular a Generalized closed sets in neutrosophic topological spaces, IOSR Journal of Mathematics, 2019, 15(02), 11-18.

[4] Jayanthi, D., On Generalized closed sets in neutrosophic spaces, International Conference on Recent Crends in Mathematics and Information Cechnology, 2018, March, 88-91.

[5] Pushpalatha, A. and Nandhini, C., Generalized closed sets Mia neutrosophic topological spaces, Malaya Journal of mathematik, 2019, 7(1), 50-54.

[6] Salama, A.A. and AlSlowi, S.A., Neutrosophic set and Nuetrosophic topological space, IOSR Journal of

Mathematics, 2012, 31-35.

[7] Santhi, R. and Udhayarani, N., N-Closed set in Neutrosophic Copological Spaces, Neutrosophic Sets and Systems, 2016, 12, 114-117.

[8] Wadei Al-Omeri and Saeid Jafari, On Generalized Closed Sets and Generalized Pre-Closed in Neutrosophic Copological Spaces, Mathematics MDPI, 2018, 7(1), 01-12.

[9] Wali, R.S. and MiMekananda DemSre, On pre generalized pre regular Weakly Closed sets in Copological spaces, Journal of Computer and Mathematical Sciences, 2015, 6(2), 113-125 Zadeh, L.A., Fuzzy Sets, Information and