Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 8s (2025)

A Study of the Population Ecology of Asian Elephants in using the Logistic Growth Model

[1*]Ahamed Amani M, [2]Dr. Manoj Kumar Singh
[1*]PhD Scholar, Department of Mathematics and Statistics Banasthali Vidyapith, Rajasthan, India
[2]Assistant Professor Department of Mathematics and Statistics Banasthali Vidyapith, Rajasthan, India
Corresponding author: ahamedamani@gmail.com

Article History:

Received: *28-10-2024*

Revised: *18-11-2024*

Accepted: *19-12-2024*

Abstract:

India is home to almost 60 percent of the world's Asian elephant population. However, with fewer than 52,000 Asian elephants left, they are listed as endangered species on the IUCN (International Union for Conservation of Nature) Red List. As keystone species, elephants are vital in maintaining ecosystem balance and require careful management. Estimating the population of these endangered animals in India is challenging due to limited resources. The optimal approach to model the population of Asian elephants in India using available data is the logistic model. This model will provide the population count of Asian elephants in India and help establish the boundaries caused by the assumptions of the logistic model. Significantly, it will also help predict whether the population of Asian elephants in India has stabilized or is likely to do so, shedding light on the efforts taken by the Indian authorities in the past decades.

Keywords: Population ecology, Mathematical Ecology, Evolution, and Social Sciences, Asian elephant’s population, logistic model.

1. Introduction

What do Asian elephants, pandas, rhinos, and gorillas have in common? They are all endangered animals(Williams 2020). A species is considered endangered when not many exist anymore.

Those of us who have seen these mammoth elephants in the wild are unlikely to ever forget the experience, with their enormous padded feet that carry them silently through the forest, huge ears that help dissipate heat, and dexterous trunks that are capable of tearing down a tree or gracefully picking up a peanut. Elephants are a significant and enduring part of human values and experience of the earth(Water 2022). Asian elephants, in their daily wanderings, disperse seeds in their dung from the 200 plant species they eat (Stokstad 2017). Those seeds grow into trees that revitalize forests, giving us oxygen to breathe and take in carbon dioxide. Asian elephants live in tightly-knit family groups where the female works together to raise young and accumulate social and ecological knowledge passed from generation to generation. Elephants maintain complex social relationships, a cognitively highly demanding task that puts them on par with other intelligent animals like bottlenose dolphins and chimpanzees. Their intelligence is legendary, and they are considered icons of wisdom across human cultures(Water 2022). They are a charismatic species that evoke strong emotions in people. The elephant is a keystone species that is pivotal in maintaining ecosystem equilibrium. Therefore, it necessitates meticulous management. Furthermore, these creatures possess substantial economic significance and are crucial in shaping the habitat.

Despite their immense intrinsic and ecological value, we have lost half of the world's Asian elephants in less than 75 years, and fewer than 50,000 now remain relegated to tiny habitat patches(Hannah 2022). If we do not act now, we may soon have to deal with their extinction and its ecological and cultural consequences. Although poaching is a threat to Asian elephants, the destruction of their habitat is a far greater threat to overlook. Asian elephants once roamed 9 million square kilometers from modern-day Iraq east to China's Yancey River and south to the island of Sumatra(People's trust for endangered species n.d.). Now, they are squeezed into just 5 percent of the historical territory and are extinct in the Middle East and most of China(People's trust for endangered species n.d.). In some areas, elephants are enslaved and forced to destroy their forest habitat- a twisted fate for one of the world's most intelligent creatures. Due to skyrocketing human population growth that has led to the clear-cutting of millions of hectares of forest, scientists say that Asian elephants will lose half of their remaining range before 2070 if they are not protected(Rahul De 2021). Moreover, because of their shrinking habitat, Asian elephants are increasingly coming into conflict with humans, forced to search for food in human landscapes. The likely dispersal area of Asian elephants overlaps with regions occupied by 55.25 million people, including 6.07 million in highly appropriate habitats. Due to the significant number of people living in potentially suitable elephant habitats, it is crucial to implement proactive measures to mitigate human-elephant conflict (HEC) in high-risk areas like India and Malaysia (Haixia Xu 2024).

Wildlife corridors, the vital links that unite diverse elephant habitats across India, enable these magnificent creatures to migrate and socialize. Regular monitoring of these corridors is not just important; it's indispensable. It provides insights into elephants' behavior, helps us understand their needs, and allows us to address any human-induced risks that could jeopardize their future. A study by Avinash Krishnan's team found that elephants heavily use certain areas of the park, especially those that cover natural habitats rather than human-dominated land. However, these areas are under high human pressure, showing the need for ongoing efforts to preserve their function as elephant corridors(Avinash Krishnan 2023).

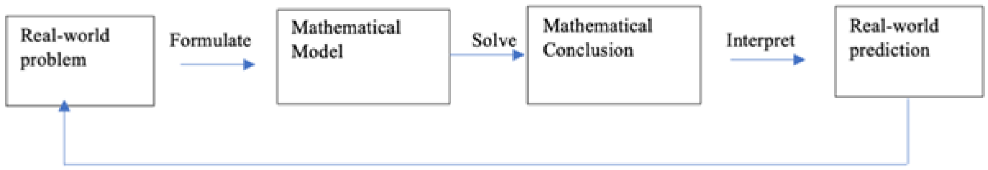
We can no longer attribute species extinction to natural causes such as earthquakes or droughts. Today, many plants and animals are becoming endangered or extinct because of habitat destruction, over-harvesting, and poaching. It is vital to start repairing the damage done to nature. Animal and plant extinction can ruin the ecosystem and reduce biodiversity. All creatures are part of an ecosystem, and they all help humans in some way. For example, over 50% of the medicines currently used are derived from a natural product made from animals or plants. By losing biodiversity, we are losing the chance to discover a new medication that could save the lives of millions of people each year(Veeresham 2012). Ullas Karanth, the director for Science-Asia at the Wildlife Conservation Society in Bronx, New York, believes that modern science demonstrates the urgent need to assist in wildlife recovery for our benefit(Karanth 2018).

The area of study that analyzes changes in population over time is known as population dynamics. In this case, our focus is on examining population trends among Asian elephants in India.To understand the process by which the population of any species either increases or decreases or is static, the three main factors to be considered are fertility, mortality, and migration. Migration may become less significant in the case of endangered animals, as there is already a reduced population, and it will make little difference in the total population. As population dynamics work on a system-based approach, we can categorize the study into two components. The first component quantitatively describes changes in population size and growth or decline for a particular organism. The second component is investigating the biological and physical processes causing these changes. We are to rely on mathematical modeling through differential equations to achieve this.

The study of animal populations involves creating mathematical models for different species living in various environments. Environmental and physical parameters are crucial in developing precise mathematical models for various animal and microbial populations. These models often involve non-linear differential and difference equations, and they consider factors such as growth, reproduction, decay, death, and migration between different domains. In his paper, Vinod P. Saxena examines interacting and non-interacting living beings while considering these essential factors. He uses cutting-edge mathematical means to study and predict environmental and topographical impacts while considering population dispersion in specific scenarios(Saxena 2023).

Differential equations are the best tool for modeling such a system when discussing changes. Vito Volterra and Alfred Lotka independently developed differential equations to describe species dynamics(Levin 2021). The most critical application of differential equations is modeling various physical and natural phenomena. Any engineering or social science problem can be solved by formulating it into a mathematical representation called a model. Modeling is about setting up a model to solve it mathematically and interpreting the results. The purpose of a mathematical model is to understand a specific phenomenon and to make predictions about the behavior or outcome of a system, event, or quantity.

The flow chart below illustrates the modeling process used in this study to model the population of Asian elephants in India.



Through the mathematical models generated with the ideas of population dynamics, we can quickly establish the equilibrium state and the stability of these models as they are critical knowledge of the system, which helps make predictions more accurate. In some cases, we might use the idea of a difference equation coupled with a differential equation, as the data obtained is more likely to be discrete. This is the case of the population data of Asia elephants; they are discrete and limited. Therefore, the focus of this paper is to apply the logistic growth model to the available data on Asia elephants and establish the limitations of the logistic model and hence indicate the error made in the inference of the specific form of the function that is used to calculate the population of the Asian elephants in India. Through this model, we can forecast the population trends of Indian Asian elephants and analyze the population patterns.

The von Bertalanffy growth function (VBGF) is a special case of the generalized logistic function used to model the growth of a time series. It is named after the Austrian biologist Ludwig von Bertalanffy. The VBGF is particularly useful in modeling the growth of organisms over time and has been applied in various fields such as ecology, fisheries science, and animal science. Ayhan Yilmaz's team, from the Department of Animal Science at Siirt University in Turkey, has conducted research utilizing the VBGF to model the mean length from age in animals. By applying this growth curve, the team aims to provide more accurate predictions of animal growth patterns, which can affect animal husbandry, wildlife management, and conservation efforts(Ayhan Yilmaz 2018).

The logistics and generalized logistics functions are examples of sigmoidal functions characterized by their S-shaped curves. These functions are widely used in various disciplines, such as economics, sociology, physics, medicine, and biology, to model growth, diffusion processes, and other phenomena exhibiting a characteristic S-shaped development pattern.

2. Basic Mathematical technique using differential equations

We first need to introduce some variables and relevant terms to model population growth using a differential equation. The variable ‘t’ represents time. The unit of time can be hours, days, weeks, months, or even years. Any given problem must specify the unit used. The variable P represents the population. Since the population varies over time, it is understood to be a function of time. Therefore, as a function of t, the notation P represents the population over the given time period (ie)P(t). Suppose P(t) is a differentiable function. In this scenario, the first derivative, dP/dt, shows the immediate rate of change of the population over time and should be directly related to the actual population size, assuming that population growth depends only on the initial size and not other factors.

$$\frac{dP}{dt} = kP \qquad (1)$$

It is clear from the above differential equation that the larger the population, the larger the rate at any given time. All species have a population growth curve based on the above simple modeled differential equation, either increasing or decreasing or static after reaching a particular time frame.

Let's consider the population P(t) with birth rate β and death rate δ. The birth rate β represents the number of births per unit of population per unit of time, while the death rate δ represents the number of deaths per unit of population per unit of time. It's reasonable to assume that birth and death rates may depend on the population size or the time interval. On the other hand, nothing restricts the model from having β and δ as constants. Furthermore, it is a reasonable dependency because, for example, the birth rate may decrease as the population increases. After all, a food restriction or overpopulation could make people less likely to have children. To set up a population model, we ask ourselves the same question when trying to derive a differential equation: how does the independent variable change? When the dependent variable changes. In other words, how does P change in some small interval of time Δt? Similarly, we will call the slight shift in population ΔP. Now, it is clear to write the following equations.

$$\Delta P = \beta P \Delta t - \delta P \Delta t = (\beta - \delta) P \Delta t \text{ (i.e.)}$$

$$\frac{\Delta P}{\Delta t} = (\beta - \delta) P \text{ (i.e.)}$$

$$\frac{dP}{dt} = (\beta - \delta) P \qquad (2)$$

The above equation (2) can be said to be the “Master” population equation from which other population models are derived.

Exponential model:

$$\frac{dP}{dt} = kP$$

where β and δ constants

Logistic model:

$$\frac{dP}{dt} = kP(M - P)$$

where δ constants and β decrease linearly with P (3)

Now, let us examine the logistic model more closely. The logistic equation is a model for population growth. Furthermore, despite its reasonably simple appearance, it works very well as a model for realistic situations, such as the growth of a country's population or any species over a significant period. The population's rate of change is expressed as the product of the population and the difference between some number M and the population P as expressed in equation (3). This number

M can be interpreted as the maximum sustainable level for the population over a long period. In addition, the fact that the expression on the right of equation (3) is a product that explains that the population grows more slowly when it is small if we have P approaching zero, and it also grows slowly when M-P is small, so the population is closed to its maximum capacity. We want to describe how the logistic equations result from the general population equation's birth and death rate assumptions (2). If we assume that we have a constant death rate and a birth rate that is not constant but somewhat decreases linearly with the population, we have the following equation.

Death rate: $\delta = \delta_0$ (constant)

Birth rate: $\beta = \beta_0 - \beta_1 P$(linearly decreases with P)

This is a reasonable assumption. One could imagine that when a population increases, it may be less desirable for the population to reproduce or for reasons like food supply may become more limited, or the habitat becomes less suitable due to encroachment or other factors which would cause the birth rate to decrease with the increase of population.

Substituting these assumptions in equation (2), we get the following equations:

$$\frac{dP}{dt} = (\beta - \delta)P \quad \text{(i.e.)}$$

$$\frac{dP}{dt} = (\beta_0 - \beta_1 P - \delta_0)P$$

By manipulating the above equation, we can obtain the logistic equation of the form:

$$\frac{dP}{dt} = \beta_1 P\left(\frac{\beta_0 - \delta_0}{\beta_1} - P\right) \tag{4}$$

Upon comparing with equation (3), we get $\frac{\beta_0 - \delta_0}{\beta_1} = M$ and $\beta_1 = k$

Moreover, further rewriting equation (3) can be put in the below form:

(i.e.)

$$\frac{dP}{dt} = kMP\left(1 - \frac{P}{M}\right) \tag{5}$$

'M' in the logistic model is the maximum population capacity. M's value represents the population with equal birth and death rates. If the rates are equal, the population will not change. Thus, as P increases and gets closer to M, the birth rate becomes more comparable to the death rate, and we find the population gradually decreases as it approaches and stabilizes at population level M. This can be seen algebraically:

At maximum population capacity M, we have birth rate = death rate. (i.e.)

$$\delta_0 = \beta_0 - \beta_1 P$$

From which we can obtain

$$P = \frac{\beta_0 - \delta_0}{\beta_1}$$

The value of P is the value that makes the death rate equal to the birth rate, and we can see this in equation (4) is the same as 'M,' which is the maximum capacity of the logistic model.

a. The verhulst-Pearl logistic growth model

Equation (5) is the logistic law of population growth described by Verhulst-Pearl as below:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \qquad (6)$$

Where N is the number of individuals in the population, r is the intrinsic rate of change, and K is the environment's carrying capacity.

We must equate the derivative to zero to obtain a differentiable function's maximum, minimum, or inflection point. In other words, on equating

$$\frac{dN}{dt} = 0 \qquad \text{(i.e.)}$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) = 0$$

Instantly we notice that the derivative will be zero at N =0 and N = K, which are the solution to the differential equation, called the equilibrium solutions. By analyzing the derivate of N, we can obtain the nature of the solution to the differential equation (6). The population increases when N's derivative is positive, which happens when the derivative takes N's value between 0 and K. On the other hand, the population stays at the same level of exponential growth when N = K and starts declining otherwise as N becomes negative. We also observe that the solution tends towards equilibrium at K; hence the solution N=K is stable.

Rewriting equation (5) and separating the variables, we obtain:

$$\frac{M}{P(M-P)}dp = kMdt$$

Moreover, by integrating, we obtain the following results:

$$P(t) = \frac{e^{kMt}M}{e^{kMt} - e^{c}}$$

'c' in the integration can be calculated using the initial condition. Further, simplifying the solution to the form as below:

$$P(t) = \frac{M}{1 - e^{c-kMt}}$$

where $M = \frac{(\beta_0 - \delta_1)}{\beta_1}$ and k = β_1, and β_o , β_1 and c are constants. Mathematically, as time increases, the population reaches a finite limit. (i.e.) P(t) approaches M as t approaches infinity, and this value of M in terms of the birth and death rate is referred to as the limitation of the population.

b. A logistic model described with the help of discrete data

In dynamic systems, we often encounter P(t) and P_t, where P is a random variable, and t is the time. If 't' is expressed as P(t), it generally denotes a continuous function of 't.' On the other hand

If 't' is expressed as P_t, it denotes a discrete function. The concepts of discrete time can be explained by the method of iteration. Iteration means repetition. An iterative method is a repetitive process that generates a sequence of outcomes. It may be difficult sometimes to find the value of the population P at a given period, and population-related data appears in discrete form, so it is challenging to find a continuous set of data for the population change in a specific time interval. The logistic equation explains the evolution of population changes over time as a continuous stream of data within a specific time interval; however, many species reproduce during one particular season or period, eventually making the population change as a discrete data set. For this reason, it may be better to describe the population change as a difference equation, expressing the change as a difference between time intervals.

A difference equation relates the independent variable, dependent variable, and its successive differences. For example, if P_n is the population of a species this year, and P_{n+1}, is the population of the following year, then the simplest form of expressing the relation is given below:

$$P_{n+1} = r\, P_n \qquad \text{where r is the growth rate.}$$

Through this assumption, we can approximate the population of the species. However, the population is bounded by the physical limits of its surroundings, so the above difference equation and be modified to show the limitation as

$$P_{n+1} = r\, P_n\, (1 - P_n)$$

The difference equation's solution can be obtained using an iterative method, which is much easier to solve than a differential equation using the traditional way. Moreover, the data obtained are discrete, so it is easy to represent them in discrete form.

3. Statement of the Problem

India has the most significant number of Asian elephants, estimated above 25000 or nearly 60 percent of the species' population. Elephants are regarded as keystone species because they are essential to the health of our ecosystem. Keystone species are critical to the ecosystem's functioning, many of which are necessary for other species' survival. In addition, elephants are among the most intelligent animals on the planet.

Elephants are the largest terrestrial animals and may be found all over Asia and Africa. Despite only being native to Africa and Asia, elephants have a profound cultural and symbolic importance worldwide. In addition, elephants are essential in helping to shape their habitat as they directly impact the composition and density of the forest, spread seeds, and change the overall landscape. They make openings and gaps in the canopy in the tropical forest, which promote the regeneration of trees and create pathways for other smaller animals to use. Many plant species in the woods of Asia and Africa require the digestive system of an elephant for their seeds to germinate. Elephants are needed to disperse the seeds of nearly one-third of the tree species in the forest of central Africa.

Elephants require large tracts of land to meet their ecological needs, which include food, water, and space. A typical elephant can feed up to 18 hours daily and eat hundreds of pounds of plant material. As a result, as they lose their habitat, they frequently clash with humans over resources. Organizations worldwide have worked for decades to save these animals and secure their existence because their function cannot be replaced by or carried out by any other species(Douglas-Hamilton 2024).

Human expansion into elephant's habitat, along with agricultural development and infrastructure such as road canals and fences, has significantly disoriented them. In addition, elephants are losing their habitat in historic migration routes. Asian elephants are currently most at risk from wildlife crime, particularly poaching for the illicit ivory trade, and are highly at risk of losing their habitat due to human encroachments. The International Union of Conservation of Nature counts Asian elephants.We can prevent elephant extinction by reducing conflict between elephants and humans, increasing poaching preventive initiatives, increasing awareness of stopping the illegal ivory trade, lowering the demand for elephant ivory, and protecting elephant habitats.

By 2015, Asian elephants had lost over 60 percent of their habitat in India. Today, the known elephant range is much smaller. How many elephants remain and precisely where they live needs to be clarified. Collecting this information will help focus on the conservation effort in an area where elephants are in danger. Based on the limited resources available on the population of Asian elephants in India, we can, to some extent, formulate a mathematical model through which we can predict their population and estimate the trend at which these animals are either decreasing or increasing in number. This study also aims to evaluate the error in predicting the population of endangered species, especially Asian elephants, as the available data is limited and to correlate with the actual data and see with the help of the derived mathematical model any changes reflecting if authorities made efforts to improve conditions of Asian elephants in India.

4. The Asian elephant population in India

The official data used in this study was collected over nearly 40 years, with a census done in a gap of every four to five years and entirely based on the information on the Asia elephant population in the Indian subcontinent. As per the International Zoo yearbook, the population of Asia elephants in 2019 was estimated to range from 48,323 to 51,680 individuals. Table 1 shows the population of Asian elephants in India between 1980 and 2018(Elephant population (AfESG &AsESG) 2019 and Swaroop 2021).

Year	Population
1980	15627
1985	18975
1989	20862
1993	15604
1997	25877
2002	26413
2007	27694
2012	29391
2017	27312
2021	29964

Table 1: Population of Asian elephants in India between 1980-2021

Plotting the years (1980 to 2021) on the x-axis and the actual population of the Asian elephant in India on the y-axis, a simple graph in Figure 1 reveals an encouraging trend. The graph indicates exponential growth, a testament to the success of conservation efforts, with a stable population of nearly over 26,000.

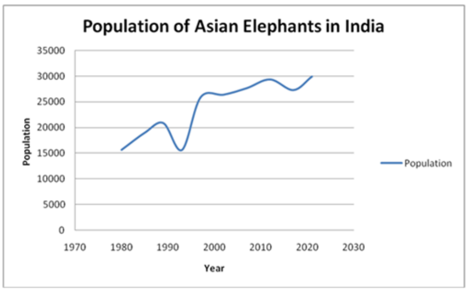


Figure 1: The graph indicates the exponential growth of Asian elephants in India.

5. Mathematical Model: Logistic Model

The population at time t is represented by P(t), and the change in population size is denoted by dP/dt or P′(t). We will assume that the population growth rate depends solely on the population's size. The aforementioned assumption has potential errors; however, in our context, it directly concerns the population estimation of the endangered Asian elephant. Consequently, the population count must be commensurate with the size of the population. As we proceed further, we can also attempt to estimate the error based on this assumption and see how much deviation that logistic model produces.

Consider the logistic model described in equation (6), where P is the population.

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \qquad (7)$$

We need to address two primary questions to show that the model described by the differential equation is logistic.

1.To get a good fit, what should be the parameter value of 'r' and 'K'?

2.And to identify the data to be a reasonable set to fit the logistic model

6. A logistic model with a difference equation

We have a discrete data set of the population of Asian elephants between the years 1980 and 2021(Hannah 2022). Hence, we bring in the idea of comparing the logistic differential equation with the difference equation, where we can find the difference in the population between years and compare it with the right-hand side of the equation (7). As the data is discrete, we use the previous value from the system to evaluate the following values. Equation (7) in terms of differences can be expressed as below:

$$P(t+1) - P(t) = rP\left(1 - \frac{P}{K}\right)$$

where t and t+1 are the successive time intervals(i.e.)

$$\frac{\Delta P}{P} = r\left(1 - \frac{P}{K}\right) \qquad (8)$$

Upon rearranging equation (7), we find that the ratio $\frac{\Delta P}{P}$ in the above equation represents a linear equation in P. To test the logistic behavior, we calculate the difference in the population for two successive available data points and use these differences against the corresponding function value. Table 2 shows the ratio of difference in the population for successive time periods.

Ratio calculations

$a_1 = \frac{P(1985)-P(1980)}{P(1980)}$	0.214245
$a_2 = \frac{P(1989)-P(1985)}{P(1985)}$	0.099447
$a_3 = \frac{P(1993)-P(1989)}{P(1989)}$	-0.25204
$a_4 = \frac{P(1997)-P(1993)}{P(1993)}$	0.658357
$a_5 = \frac{P(2002)-P(1997)}{P(1997)}$	0.020713
$a_6 = \frac{P(2007)-P(2002)}{P(2002)}$	0.048499
$a_7 = \frac{P(2012)-P(2007)}{P(2007)}$	0.061277
$a_8 = \frac{P(2017)-P(2012)}{P(2012)}$	-0.07074
$a_9 = = \frac{P(2021)-P(2017)}{P(2017)}$	0.097100

Table 2:Ratio showing the population difference for two successive periods.

Considering the ratio value with the corresponding P(Population) value, we have table 3 formed below:

a -value	P(t)
0.214245	15627
0.099447	18975
-0.25204	20862
0.658357	15604
0.020713	25877
0.048499	26413
0.061277	27694
-0.07074	29391
0.097100	27312

Table 3: Ratio with the corresponding population to estimate the linearity

If we can show that the plots of the ratio value against the P value are linear, then the model represented by the equations (7) is reasonably logistic.

Figure 2 shows the least-square approximation graph using the data from Table 2. It offers the linear approximation line given in Equation (9).

$$y = -0.00003x + 0.7115 \quad (9)$$

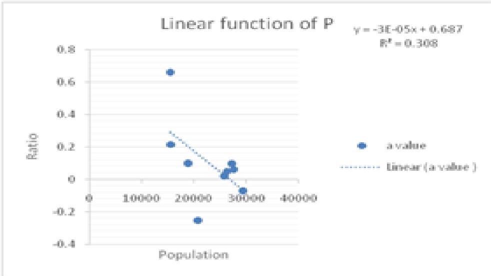


Figure2: Least-square approximation graph to show the linearity using the ratio(a) from Table 3

It is seen that the ratio value ‘a’ can be obtained at various population levels P(t) at a given time t. Figure 2 also indicates that, except for a few data points, most estimated values are near the least square approximation line. Hence, our assumption for equation (7) is reasonable that the model shows a logistic behavior.

7. The solution of the model

a. Find the value of the parameters ‘r’ and ‘K.’

From the least square approximation (Figure 2), we have the straight-line approximation equation (9) through which we can calculate the value of y_1 and y_2 by substituting P(1980) and P(1985)

(i.e.) $y_1 = -0.00003(15627) + 0.711 = 0.24219$ and

$y_2 = -0.00003(18975) + 0.711 = 0.14175$

The 1980, 1985, and 1989 data are used in equation (9). We have the following equations.

$$r\left(1-\frac{15627}{K}\right)=0.24219$$

$$r\left(1-\frac{18975}{K}\right)=0.14175$$

By solving the above equations, we evaluate the value of ‘r’ and ‘K’ as

$r = 0.711$ and $K = 23700$

Hence, we complete the logistic model with the estimated values of ‘r’ and ‘K.’

$$\frac{dP}{dt}=0.711P\left(1-\frac{P}{23700}\right) \quad (10)$$

The values for r and K obtained are positive and have biological significance. It has been observed that P′ is approximately equal to rP when P is small, and P′ equals 0 when P is near K. To put it simply, when P is small, the population undergoes exponential growth. When P is near K, the population hardly changes. By solving the equations, we find that K equals 23700, which estimates the population's carrying capacity, representing the population that size available resources can continue to support, with a per capita intrinsic growth rate value of r= 0.711. We can obtain an average value of K by choosing different years.

b. Obtain a Particular solution to the Logistic Model.

Equation (10) can be rewritten as

$$\frac{dP}{dt}=0.711P-0.00003P^2 \quad (11)$$

We can solve the above equation using the integration technique. Equilibrium for the differential equation (10) is defined as the value $x_\propto$such as $f(x_\propto)=0$, which helps estimate the population's carrying capacity.

Particular solution to the Logistic Model

And using the technique of variable separable

$$\int \frac{dP}{P(0.711-0.00003P)} = t + c$$

Further, the integral is split as

$$\frac{1}{P(0.711-0.0003P)} = \frac{1}{0.711}\left(\frac{1}{P} - \frac{0.00003}{0.711-0.00003P}\right) \quad \text{(i.e.)}$$

$$\frac{1}{0.711}\int\left(\frac{1}{P} - \frac{0.00003}{0.711-0.00003P}\right)dP = t + c \qquad (12)$$

When t=0, the corresponding initial population in 1980 is used to calculate the integral constant

(i.e.) $P_0=15627$ at $t = 0$

$$c = \frac{1}{0.711} ln\left(\frac{15627}{0.711-0.00003(15627)}\right) = 15.576$$

Using the ‘c’ value

$$\frac{1}{0.711} ln\left(\frac{P}{0.711-0.00003P}\right) = t + 15.576$$

Solving the integral for P, we have the final equation of the logistic model.

$$P(t) = \frac{23700}{1+0.5165e^{-0.711t}} \qquad (13)$$

As mentioned, an equilibrium for the differential equation (10) is defined as the value $x_\propto$such as $f(x_\propto)=0$.

$$f(x_\infty) = 0.711x_\infty\left(1 - \frac{x_\infty}{23700}\right) = 0$$

We see that if we take the solution's limit as ‘t’ tends to infinity, we obtain P(t) = 23700, which is the population's carrying capacity.

c. Comparing the Logistic Model with the actual data

Once we have solved the differential equation (10) that forecasts the population growth of Asian elephants in India, we can compare the actual data with the calculated outcomes. This will allow us to determine how well the logistic model constrains the population.

d. Estimating error function

Based on the provided year and the difference, we can confidently estimate a function to supplement the solution of the specified differential equation (10) and effectively minimize calculation errors.

In other words, we can establish a function Y(t) as the function in years, which has to be adjusted with the differential equation’s (10) solution to get the closest approximation to the actual population.

Thus the equation

$$P(t) = S(t) + Y(t) \tag{14}$$

where S(t) is the solution to the differential equation (10),and Y (t) is the adjusted function to give a better approximation for the population of Asian elephants.

From equation (14), we would like to trace back to the actual differential equation to find the missing error function, which led to Y(t) upon integration. To achieve this, we have to differentiate the equation (14) with respect to ‘t.’

A practical analysis reveals that the model is most suitable as an approximation and that rather than P’=f(P), we should study a differential equation in the condition (Brauer 2010).

$$\frac{dy}{dt} = f(y) + h(y) \tag{15}$$

The term h(y) denotes the error that arises when we assume a specific form f(y). Although it is challenging to determine h(y) explicitly, we can estimate it by retracing our steps from the predicted solution to the original differential equation.

8. Limitationsbased on the assumption and further discussion

a. Form of the differential equation

The error in the logistic model was based on the assumption of no external influences on the population. A realistic approach would be to study the differential equation taking the form below.

$$\frac{dy}{dt} = f(y) + h(y)$$

instead of $x' = f(x)$, where h(y) indicates the error in our assumption.

As the function h(y) cannot be found explicitly. However, the below theorem throws some insight into its construction of it.

Theorem: Let x_∞ be an asymptotically stable equilibrium of $x' = f(x)$ with $f'(x_\infty) < 0$. Then (i) If $h(y)/(y - x_\infty) \rightarrow 0$ as $y \rightarrow x_\infty$, and if $|y(0) - x_\infty|$ is sufficiently small, the solution y(t) of $y' = f(y) + h(y)$ tends to $x\infty$ as $t \rightarrow \infty$, i.e., x_∞ is an asymptotically stable equilibrium of $y' = f(y) + h(y)$. (Brauer 2010)
Based on the theorem, we have

$$\lim_{y \to x_\infty} \frac{h(y)}{y - x_\infty} \rightarrow 0$$

and h(y) approaches 0 much faster than $y - x_\infty \rightarrow 0$. Therefore, h(y) has to be a function of $(y\text{-}x_\infty)$. The value of the function $h(y\text{-}x_\infty)$ can be approximated with the help of the adjusted equation Y(t) from equation (14), and the error can be minimized by choice of Y(t) as h(y) is the error function that has to be added to the original differential equation. So, it remains to prove that h(y), which is a function of $(y\text{-}x_\infty)$, can be obtained from Y(t), and also, it has to be a function of the higher power of the term $(y\text{-}x_\infty)$.

b. Findings:

Using the logistic model, we have derived the essential population growth of the Asian elephants in the Indian subcontinent, as shown below:

$$\frac{dP}{dt} = 0.711P\left(1 - \frac{P}{23700}\right)$$

Rames K's team's research findings reveal that the elephant population in India is relatively stable, with an estimated range of 25,000 to 30,000 individuals(Ramesh K. Pandey 2024). The differential equation (10) demonstrates asymptotic stability, with a carrying capacity of 23700 for the population. Based on the initial assumptions in the population model and the associated errors, it is reasonable to infer that various external factors, such as climate change, habitat loss, human-wildlife conflict, and poaching, may have significantly influenced the model's deviation. Moreover, the data considered in our study spans from 1980 to 2021, and Rames K's team's stability was established more recently in 2024, indicating a relatively short period of stability assessment. Therefore, it is well within our reach to suggest that the population of Asian elephants is showing signs of heading toward stability based on the estimation given by our model.

One reason for achieving this stability is the country's development and the implementation of institutional, policy, and legal reforms that have enabled elephant conservation in India. The Indian Wildlife Protection Act (1972) protects Elephas maximus under Schedule I and Part I. With public support for wildlife conservation and political backing, India successfully balances coexistence and conflict regarding elephants(Ramesh K. Pandey 2024).

As this basic model is constructed assuming no external factors influence the model other than the initial size, it might tend to show an error compared to the actual population of Asian elephants in India. However, the error is minimized by constructing a suitable function Y(t) and adding it to the fundamental differential equation obtained from the logistic model. Thus, we have the adjusted logistic equation as below:

$$P' = 0.711M\left(1 - \frac{M}{23700}\right) + Y'$$

where M = P-Y and Y (t) is the adjusted function

As we gather more data, finding the function Y(t) with greater accuracy becomes easier, minimizing the error. According to the theorem, the constructed function has a higher power of the term $(y-x_\infty)$. Using this adjusted differential equation, we have minimized the error between the actual and estimated population of Asian elephants in India. Therefore, the more accurate the function Y(t) we predict, the more precise the population count of Asian elephants in India can be achieved. This projected population trend helps authorities better understand the usage of resources and efforts required to protect these endangered animals. Also, it gives a basic insight into the trends of the population of Asian elephants.

REFERENCES

[1] Avinash Krishnan, Nicholas Warren , Upasana Ganguly , Ekadh Ranganathan and Dilip Kumar. 2023. "Road for Elephants: Elephant Use of the Karadikkal-Madeshwara Corridor, Southern India." Gajah 56 (2023) 4-9 https://doi.org/10.2305/DIHX2111

[2] Ayhan Yilmaz, Ferda Karakus, Mehmet Bingöl, Baris Kaki, Gazel Ser. 2018. "Effects of some factors on growth of lambs and the determination of growth curve models." Indian Journal of Animal Research https://doi.org/10.18805/ijar.B-815

[3] Brauer, Fred. 2010. "Mathematical Models in Population Biology and Epidemiology." 20-23 Springer.

[4] D. Jathanna, K.U. Karanth, N.S. Kumar, K.K. Karanth, V.R. Goswami. 2015. "Patterns and Determinants of Habitat Occupancy by the Asian Elephant in the Western Ghats of Karnataka, India." elsevier https://doi.org/10.1371/journal.pone.0133233.

[5] Douglas-Hamilton, Dr. Iain. 2024. WHY CARE. https://worldelephantday.org/about/elephants

[6] Elephant population (AfESG &AsESG, 2019) – processed by Our World in Data. “Number of Asian elephants” . Elephant population (AfESG &AsESG, 2019).

[7] https://ourworldindata.org/grapher/number-of-asian-elephants?tab=chart&time=earliest..2017

[8] Haixia Xu, Luguang Jiang,Ye Liu. 2024. "Ecological Informatics." Elsevier https://doi.org/10.1016/j.ecoinf.2024.102518

[9] Hannah, Ritchie. 2022. The state of the world's elephant populations. 12 1. https://ourworldindata.org/elephant-populations#:~:text=For%20the%20Asian%20species%2C%20this,on%20the%20IUCN%20Red%20List.

[10] Karanth, K. Ullas. 2018. "Reconciling Rights of Individuals with Rights of Wild Species." Ecology, Economy and Society–the INSEE 87-90, https://doi.org/10.37773/ees.v1i1.21.

[11] Levin, Simon A. 2021. "Mathematical Ecology, Evolution, and the Social Sciences." Ecology, Economy and Society–the INSEE Journal no. 1 (2021): 5–12. https://doi.org/10.37773/ees.v4i1.401.

[12] n.d. People's trust for endangered species. Accessed January 5 ,2024. https://ptes.org/danger-of-death-for-asian-elephants/#:~:text=Asian%20elephants%20are%20the%20smaller,the%20Yangtze%20River%20in%20China.

[13] Rahul De, Reeta Sharma, Priya Davidar, N. Arumugam, Arulmozhi Sedhupathy, Jean-Philippe Puyravaud, K. Muthamizh Selvan, P.P. Abdul Rahim, Swati Udayraj, Jyotirmayee Parida, Dinesh Kumar Digal, Rajapandian Kanagaraj, Kashmira Kakati, Parag Nigam, A. Christ. 2021.

[14] "Pan-India population genetics signifies the importance of habitat connectivity for wild Asian elephant conservation,." Global Ecology and Conservation, Volume 32, https://doi.org/10.1016/j.gecco.2021.e01888.

[15] Ramesh K. Pandey, Satya P. Yadav, K. Muthamizh Selvan, Lakshminarayanan Natarajan, Parag Nigam. 2024. "Elephant conservation in India: Striking a balance between coexistence and conflicts." Integrative Conservation 1-11 https://doi.org/10.1002/inc3.38.

[16] Saxena, Vinod P. 2023. "Mathematical modelling of biological populations with and without dispersion." Taylor & Francis Volume 1 https://doi.org/10.1080/27684520.2023.2215638.

[17] STOKSTAD, ERIK. 2017. "This is amazing!' African elephants may transport seeds farther than any other land animal." Science doi: 10.1126/science.aal1023. https://www.science.org/content/article/amazing-african-elephants-may-transport-seeds-farther-any-other-land-animal.

[18] Swaroop, Brijendar. 2021. "Asian elephants (Elephas Maximus) and its Global Status." TRUMPET P R O J E C T E L E P H A N T D I V I S I O N MINISTRY OF ENVIRONMENT, FOREST & CLIMATE CHANGE 2020 https://moef.gov.in/wp-content/uploads/2021/08/Newsletter_FINAL_compressed.pdf.

[19] Veeresham, Prof. (Dr.) Ciddi. 2012. "National Library of Medicine." Journal of Advanced Pharmaceutical & Research 10.4103/2231-4040.104709. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3560124/.

[20] Water, Antoinette van de,1 , Enrico Di Minin,Rob slotow. 2022. "Human-elephant coexistence through aligning conservation with societal aspirations." Global Ecology and Conservation, Volume 37 https://doi.org/10.1016/j.gecco.2022.e02165. Williams, C., Tiwari, S.K., Goswami, V.R., de Silva, S., Kumar, A., Baskaran, N., Yoganand, K. & Menon, V. 2020. Elephas maximus. The IUCN Red List of Threatened Species 2020. https://dx.doi.org/10.2305/IUCN.UK.2020-3.RLTS.T7140A45818198.en