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Impatient Customer Queue with Single Working Vacation

A.Saila kumari[1] * K. V. S. Nagalakshmi [2]

1. .A.Saila kumari,Associate Professor,Department of Mathematics , JNTUA college of engineering,Anantapur-515002
2. Research Scholar and Lecturer in Mathematics, Govt. Degree College, Visakha Women's, Visakhapatnam

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Abstract:

This paper analyzes an infinite buffer M/M/1 queue with single working vacation in which customers arrive according to a Poisson process. As soon as the system becomes empty, the server takes working vacation. The service rate during regular busy period, working vacation period and vacation times are assumed to be exponentially distributed. During working vacation customers may renege due to impatience and the impatient timer follows exponential distribution. The steady-state probabilities and performance measures are obtained by using probability generating function. Various numerical results are presented to show the effect of the some parameters on the system performance measures.

Keywords: Queue, Single working vacation, Reneging, Probability generating function.

1 Introduction

The queueing model with server vacations has been well studied in the past three decades and successfully applied in many areas such as manufacturing and production systems, computer and communication systems, etc. Excellent surveys on the earlier works of vacation models have been reported by Doshi [2], Tian and Zhang [9], Ke et al. [4] and so on. Working vacation (WV) is one kind of vacation policy under which the server provides service at a lower speed during the vacation period rather than stopping service completely. It was introduced by Servi and Finn [7] in an $M/M/1$ queueing system.

The WV policies are classified as single working vacation (SWV) and multiple working vacation (MWV). Under the SWV policy the server enters into vacation, when there are no customers in the queue and serves at a lower rate to the . Meanwhile, he only takes one each time, and must come back to the normal working level no matter whether there are customers at the vacation ending instant. If there are customers at the vacation ends, the server begins to serve one customer at the normal rate immediately; otherwise, he will stay in an idle period. Wu and Takagi [12] generalized Servi and Finn [7] $M/M/1/WV$ queue to an $M/G/1/WV$ queue. Baba [1] analyzed a $G/M/1$ queue with MWV. Tian et al. [10] studied and $M/M/1$ queue with SWV using matrix geometric method. Li and Tian [5] generalized this study to $GI/M/1$ queue with SWV and obtained the queue length distributions at pre-arrival and arbitrary epochs using matrix geometric approach.

Impatient is the most prominent characteristic as individuals always feel anxious and impatient during waiting for service in real life. The customers' impatient acts should be involved in the study of queueing system to model real situations exactly. Intermittent operation of a service can be economically appealing whenever full time service would result in significant server idle time or would preclude the use of the server in some other productive capacity. On the other hand, having the server in operative for periods of time may increase the probability of customer loss due to reneging. An $M/M/1$ queue with customers reneging has been discussed in Haight [3]. Laxmi et al.[6] presented the analysis of finite buffer Markovian queue with balking, reneging, and working vacations. Selvaraju and Goswami [8] studied impatient customers in an $M/M/1$ queue with single and multiple working vacations. Vijaya Laxmi and Jyotsna [11], analyzed the Impatient customer queue with Bernoulli schedule vacation interruption.

This paper is an extension of the earlier of the earlier works Tian et al. [10] by including reneging. Using probability generating functions, explicit expressions of the steady-state probabilities are obtained. Various performance measures such as the average system length, average reneging rate are discussed with these performance measures, we demonstrate the parameter effect on the performance indices of the system. The rest of the paper is organized as follows. Section 2 presents description of the model. The explicit expressions of the steady-state probabilities are obtained in Section 3. A variety of numerical results are presented in section 4 followed by conclusion in Section 5.

2 Model description

Let us consider an infinite buffer single server queueing system in which customers arrive according to a Poisson process with rate λ and server can serve only one customer at a time with an exponential rate μ. The server begins a working vacation of random length V at the instant when the queue becomes empty, and vacation duration V follows an exponential distribution with parameter ϕ. During a working vacation an arriving customer is served at a rate η. When a vacation ends, if there are customers in the queue, the server changes service rate from η to μ and a regular busy period starts. Otherwise the server enters idle period, and a new regular busy period starts when a customer arrival occurs.

An arriving customer who joins the system and finds the server in WV, activates an impatient timer T, which is exponential with rate α. If the server is available during WV before the time T expires, the customer is served with η. If WV finishes before the impatient timer expires, the server switches to normal working period, and the customer is served with rate μ. Since the arrival and departure of an impatient customer without service are independent the average reneging rate of a customer is given by $n\alpha$, where n is the number of customers.

3 Steady-state probabilities

At time t, let $L(t)$ denote the number of customers in the system and $J(t)$ be the status of the server, which is defined as $J(t) = 0$, when the server is in WV and $J(t) = 1$, when server is in normal working period.

The $\{(J(t), L(t)), t \geq 0\}$ is a Markov process with the state space $\Omega = \{(j, n): n \geq 0, j = 0,1\}$.

Let $\pi_{j,n} = \lim_{t\to\infty} \pi\{J(t) = j, L(t) = n\}$, $n \geq 0$, $j = 0,1$, denote the steady-state probabilities of the process $\{(J(t), L(t)),\ t \geq 0\}$. Then, the set of balance equations are given as follows: (+)_0,0 = (+)_0,1+_1,1,

(+++)_0,1 = _0,0+(+2)_0,2,

(+++n)_0,n = _0,n-1+(+(n+1))_0,n+1;   n   2, _1,0 = _0,0,

(+)_1,1 = _1,0+_1,2+_0,1,

(+)_1,n = _1,n-1+_1,n+1+_0,n; n2. The steady state probabilities are obtained by solving the above system of equations (3)-(3) using probability generating function. Define $G_0(z)$ and $G_1(z)$ as

$$G_0(z) = \sum_{n=0}^{\infty} z^n \pi_{0,n}, G_1(Z) = \sum_{n=0}^{\infty} z^n \pi_{1,n},$$

with

$$G_0(1) + G_1(1) = 1.$$

and $G_0'(z) = (d/dz)G_0(z) = \sum_{n=1}^{\infty} nz^{n-1}\pi_{0,n}$. Multiplying equations (3)-(3) by 1, z, z^n; respectively, summing over n and rearranging the terms. We get

$$\alpha z(1-z)G_0'(z) + (\lambda z^2 - (\lambda+\phi+\eta)z + \eta)G_0(z) + \mu\pi_{1,1}z - (1-z)\eta\pi_{0,0} = 0. \quad (1)$$

In similar manner, from equations (3)-(3), we have

$$(\lambda z - \mu)(1-z)G_1(z) = \phi z G_0(z) - \mu(1-z)\pi_{1,0} - \mu\pi_{1,1}z. \quad (2)$$

From equation (1) can be written as

$$G_0'(z) - \left(\frac{\lambda}{\alpha} + \frac{(\phi+\eta)}{\alpha(1-z)} - \frac{\eta}{\alpha z(1-z)}\right)G_0(z) = \frac{\eta}{\alpha z}\pi_{0,0} - \frac{\mu}{\alpha(1-z)}\pi_{1,1}$$

for $z \neq 0$ and $z \neq 1$. Multiplying with $e^{-(\lambda/\alpha)z}(1-z)^{\phi/\alpha}z^{\eta/\alpha}$ on both sides in the above equation and integrating from 0 to z yields, G_0(z)=-_1,1k_1(z)+_0,0k_2(z) e^-(/)z(1-z)^/ z^/, where k_1(z)=_0^ze^-(/)x(1-x)^(/)-1x^/dx,

k_2(z)=_0^ze^-(/)x(1-x)^/x^(/)-1dx. Let $G_0(1)$ and $G_1(1)$ denotes the sum of probabilities when the server is in working vacation and normal busy period and define $G_0(1) = \sum_{n=0}^{\infty} \pi_{0,n}$ and $G_1(1) = \sum_{n=0}^{\infty} \pi_{1,n}$, respectively. Taking $z = 1$ in equation (3), we get

$$\mu\pi_{1,1}k_1(1) = \eta\pi_{0,0}k_2(1). \quad (3)$$

Therefore equation (3) becomes as

$$G_0(z) = \frac{e^{(\lambda/\alpha)z}}{(1-z)^{\phi/\alpha}z^{\eta/\alpha}}\left[k_2(z) - \frac{k_2(1)}{k_1(1)}k_1(z)\right]\frac{\eta}{\alpha}\pi_{0,0}. \quad (4)$$

The average number of customers in the system during a working vacation is dinotes $E[L_0]$ and define $E[L_0] = G_0'(1) = \sum_{n=1}^{\infty} n\pi_{0,n}$. From the equation(1),

$$G_0'(z) = \frac{(1-z)\eta\pi_{0,0} - (\lambda z^2 - (\lambda+\phi+\eta)z + \eta)G_0(z) - \mu\pi_{1,1}z}{\alpha z(1-z)}.$$

Take $z = 1$ and using L'Hospital rule, $G_0'(1)$ can be obtained as

$$G_0'(1) = \frac{(\lambda-\eta)G_0(1) + \eta\pi_{0,0}}{(\alpha+\phi)}. \quad (5)$$

Now we obtained $G_0(1)$ and $\pi_{0,0}$. Taking $z = 1$ in equation (2), we have

$$G_0(1) = \frac{\mu\pi_{1,1}}{\phi}. \quad (6)$$

From equation (2), $G_1(z)=\frac{\phi z G_0(z)-\mu(1-z)\pi_{1,0}-\mu\pi_{1,1}z}{(\lambda z-\mu)(1-z)}$, and using L'Hospital rule by taking $z=1$, then $G_1(1)$ can be obtained as

$$G_1(1)=\frac{\phi G_0^{'}(1)+\mu\pi_{1,0}}{(\mu-\lambda)}. \tag{7}$$

From the equations (3) and (6), $G_0(1)$ can be written as $G_0(1)=\beta\pi_{0,0}$, where $\beta=\frac{\eta k_2(1)}{\phi k_1(1)}$.

Using equation (3) and the normalizing condition i.e. $G_0(1)+G1(1)=1$, we get

$$\pi_{0,0}=\frac{\lambda=(\alpha+\phi)(\mu-\lambda)}{(\alpha+\phi)(\beta\lambda(\mu-\lambda)+\mu\phi)+\lambda\phi(\beta(\lambda-\eta)+\eta)}. \tag{8}$$

4 Performance measures

In this section, we present some important performance measures of the model. Let $E[L_1]$ denote the average number of customers in the system when the server is in normal working period. Using equation (3), and Differentiating equation (2) with respect to z and using L'Hospital rule twice, the expression for $E[L_1]$ is obtained as

$$E[L_1]=G_1^{'}(1)=\frac{\phi G''_0(1)}{2(\mu-\lambda)}+\frac{\mu\phi}{(\mu-\lambda)^2}E[L_0]+\frac{\mu\phi}{(\mu-\lambda)^2}\pi_{0,0}. \tag{9}$$

Differentiating equation (1) with respect to z and using L'Hospital rule,taking $z=1$, we have

$$G''_0(1)=\frac{2((\lambda-\phi-\eta-\alpha)E[L_0]+\lambda G_0(1))}{2\alpha+\phi},$$

substituting $G''_0(1)$ and $E[L_0]$ in equation (4) yields $E[L_1]$. Therefore, the average system length $E[L]$ is given by

$$E[L]=E[L_0]+E[L_1].$$

As the instantaneous reneging rate during WV is $n\alpha$, the average reneging rate $(R.R)$ is

$$R.R=\sum_{n=1}^{\infty} n\alpha\pi_{0,n}=\alpha E[L_0]$$

5 Numerical results

In this section, we present the effect of some parameters on the system performance measures through some numerical results. The various parameters of the model are chosen as $\lambda=1.7$; $\mu=2.5$; $\eta=1.9$; $\phi=1.5$; $\alpha=1.1$; unless they are considered as variables or their values are mentioned in the respective figures.

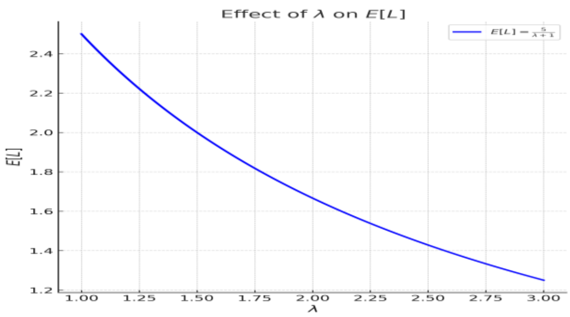

Figure 1: Effect λ on $E[L]$

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Figure 2 presents the effect of arrival rate (λ) on the average system length ($E[L]$) for different vacation rates (ϕ). $E[L]$ increase with the increase of λ for fixed ϕ and it decreases with the increase of ϕ for fixed λ as it should be.

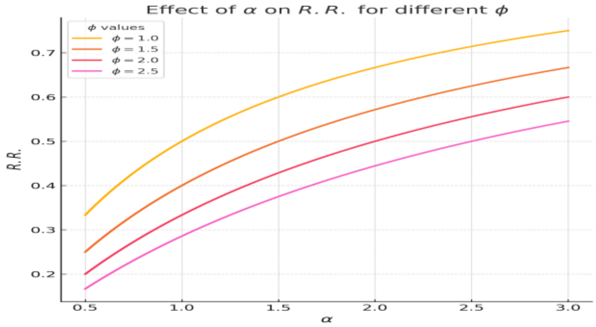

Figure 2: Effect $R.R.$ for different ϕ

Figure 2 shows the effect of α on the reneging rate ($R.R.$) for different vacation rates (ϕ). As expected in real life, $R.R.$ increases with the increase of α. In this paper we considered that, the arrived customer may renege due to impatience whenever the server is in WV. However, the server spends more time in normal working period with the increase of ϕ and hence $R.R.$ decrease with the increase of ϕ for fixed λ. We can clearly observe this case from Figure 2.

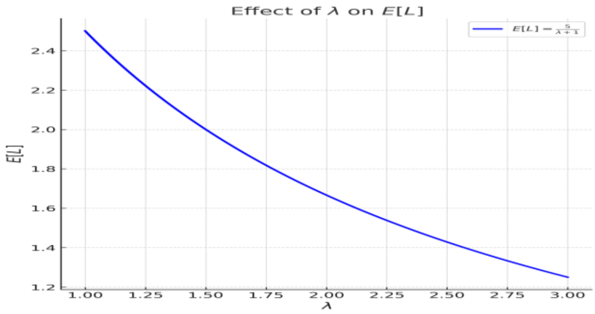

Figure 3: Effect η on $E[L]$ for different ϕ

The effect of service rates in working vacation period (η) on the average system length $E[L]$ for different vacation rates ϕ is given in Figure 3. We know that, $E[L]$ decreases with η for fixed ϕ and it increases with the increase of ϕ for fixed η as it clear from real life.

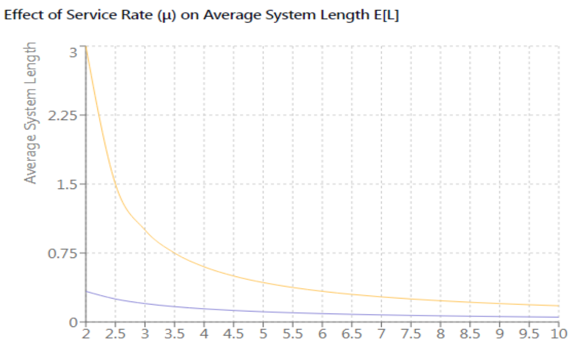

Figure 4: Effect μ on $E[L]$ for different λ

Figure 4 gives the effect of service rates in normal working period (μ) on the average system length $E[L]$ for different arrival rates λ. As explained in Figure 2, $E[L]$ increases with λ for fixed μ. Also it decreases with the increase of μ for fixed λ.

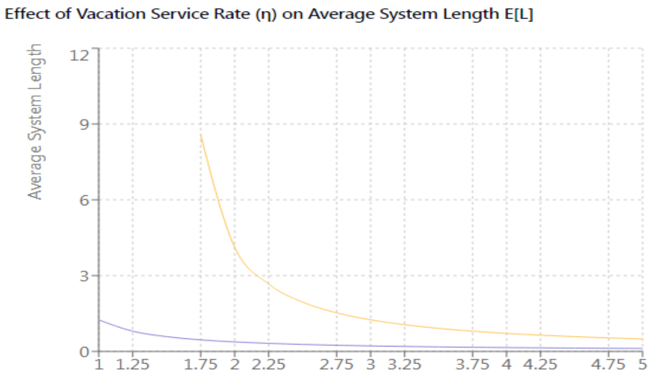

Figure 5: Effect η on $E[L]$ for different λ

Figure 5 depicts the effect of service rates η on the average system length $E[L]$ for different arrival rates λ . Here also $E[L]$ increases with the increase of λ for fixed η and decreases with increase of λ for fixed η as in Figure 4. But, the effect of service rate on $E[L]$ is more in Figure 4 when compare with Figure 5. This is due to the service rate in working vacation period is lower than the service rate in normal working period.

6 Conclusion

In this paper, we have studied an $M/M/1$ queueing system with single working vacation and customers' impatience. We have obtained the closed form expressions of the steady state probabilities. Some important performance measures of the model such as average system length and average reneging rate. The effects of some parameters on the performance measures of the system have been investigated numerically and presented through graphically. The techniques adopted in this paper can be applied to analyze models like impatient customer $M/M/1$ queue with variant working vacation.

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