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(S, d)Magic Labeling of Some Cycle Related Graphs

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Abstract:
Objective: To examine the existence of (s,d) Magic Labeling on cycle related graphs.

Methods: Let G (p,q) be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Let f:V(G)→{s,s+d,s+2d,..s+(q+1)d} and g:E(G)→{d,2d,3d…2(q-1)d} be an injective function. Then, for any u,v∈V(G) and uv∈E(G),f(u)+g(uv)+f(v) is a constant, and the function f is said to be (s, d) magic labeling. If a graph G admits (s,d) magic labeling, then it is referred to as a (s,d) magic graph.

Findings: In this paper the existence of (s,d) magic labeling in some cycle related graphs such as a Cycle graph C_(n⊙K_(1,m)) graph, n -Sunlet graph, Friendship graph Flower graph and wheel graph were found.

Novelty: The labeling of the vertices and edges is done mathematically, and this leads to the creation of a new labeling known as (s,d) magic labeling.

Keywords: Cycle graph, C_(n⊙K_(1,m)) graph, n-Sunlet graph, Friendship graph, Flower graph and wheel graph.

1. Introduction

[1,3] The graphs that are being studied are simple, undirected, finite, and non-trivial. Graph labeling has advanced significantly according to graph theory, which has a wide range of uses. In 1963, Sedl´ácek developed the first magic-type labeling. He gave a graph's edges real numbers and mandated that the total of all the labels on all the edges that are incident to a vertex must remain constant. [5] Hegde SM, Shetty S. have introduced (k, d) - arithmetic labeling of graphs. We have showed (s, d).magic labeling for the path, star, and a few standard graphs. This work established the existence of (s, d) magic labeling in certain cycle-related graphs.

[10] Let $G\ (p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: V(G) \to \{s, s+d, s+2d, .. s+(q+1)d\}$ and $g: E(G) \to \{d, 2d, 3d \ldots 2(q-1)d\}$ be an injective function. Then, for any $u, v \in V(G)$ and $uv \in E(G), f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be (s, d) magic labeling. If a graph G admits (s, d) magic labeling, then it is referred to as a (s, d) magic graph.

Several studies contribute to graph theory: [2] Farida et al. explore graph labeling, emphasizing magic covering and edge super magic labeling, revealing practical applications in secret sharing and ruler models. [4,6] Ghodasara G.V. et al. and Baskar Babujee J.et.al. introduce prime cordial labeling, presenting innovative constructions and advancements in the field. [7] K. Kavitha et al. define group magic labeling for connected graphs, focusing on specific structures like cycles with a common vertex and chains of even cycles. [8] S. K. Vaidya et al. introduce prime cordial labeling based on gcd properties, demonstrating its applicability to various graph structures such as gear graphs, helms, closed helms, and flower graphs.

2. Methodology

Definition 2.1: Cycle graph is a graph of n vertices with exactly n edges. Cycle graph contains cycle of length n

Definition 2.2: [9] The graph $C_n \odot K_{1,m}$ is obtained by attaching m leaves to each vertex of the cycle C_n

Definition 2.3: The Sunlet graph, represented by S_n, is a graph with two n vertices that is created by joining n − pendant edges to the cycle C_n.

Definition 2.4: A graph called friendship, denoted by F_n is constructed by n triangles that share a vertex.

Definition 2.5: A flower graph, or Fl_n, is a graph that is created by connecting each pendent to the helm's central vertex.

Definition 2.6: The graph $C_n + K_1$.denoted by W_n is called wheel graph, n is a number of vertices in the cycle.

3. Results and Discussion

Theorem 3.1

The cycle graph C_η is (s, d) magic labeling

Proof: Let C_η be the cycle graph. $|V(C_\eta)| = |E(C_\eta)|$= η. Let $V(C_\eta) = \{v_i; 1 \le i \le \eta\}$ and $E(C_\eta) = \{v_i v_{i+1} : 1 \le i \le \eta - 1\} \cup \{v_1 v_\eta\}$

Define the function f from the vertex set to $\{\{s, s+d, s+2d \dots . s + (|E|+1)\}$, $g: E(G) \to \{d, 2d, 3d \dots . . 2(|E|-1)d \}$

Labeling of Vertices		
Cases	Value of i	$f(v_i)$
η is even	$i = 1$	s
	$2 \le i \le \frac{\eta}{2} + 1$	$s + (2i - 3)d$
	$\frac{\eta}{2} + 2 \le i \le \eta$	$s + (18 - 2i)d$
η is odd	$1 \le i \le \eta$	$s + (i - 1)d$

Table 1: Labeling of vertices for the graph C_η

Labeling of Edges			
Cases	Value of $\dot{l}$	$g(v_i v_{i+1})$	$g(v_i v_1)$
η is even	$\dot{l} = \eta$	-	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_1))$
	$1 \leq \dot{l} \leq \eta - 1$	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_{i+1}))$	-
η is odd	$\dot{l} = \eta$	-	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_1))$
	$1 \leq \dot{l} \leq \eta - 1$	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_{i+1}))$	-

Table 2: Labeling of edges for the graph C_η

Therefore $f(v_i) + f(v_{i+1}) + g(v_i v_{i+1})$, $f(v_1) + f(v_i) + g(v_1 v_i)$ are constant equals to $2(s + (|E| - 1)d)$. Hence the Cycle graph C_η admits (s, d) magic labeling.

Theorem 3.2 The graph $C_\eta \odot K_{1,m}$ is (s, d) magic labeling

Proof: Let $V(C_n \odot K_{1,m}) = \{v_i : 1 \leq \dot{l} \leq \eta\} \cup (u_i^j : 1 \leq \dot{l} \leq \eta, 1 \leq \dot{l} \leq$m} and $E(C_n \odot K_{1,m}) =$ $\{v_i v_{i+1} : 1 \leq \dot{l} \leq \eta - 1\} \cup \{v_\eta v_1\} \cup (v_i u_i^j : 1 \leq \dot{l} \leq \eta, 1 \leq \dot{l} \leq$m}

$|V(C_n \odot K_{1,m})| = |E(C_n \odot K_{1,m})| =$ η+ηm.

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s + (|E| + 1)\}$, $g: E(G) \rightarrow$ $\{d, 2d, 3d \ldots . . 2(|E| - 1)d\}$

Labeling of Vertices				
Cases	Value of $\dot{l}$	$f(v_i)$	$f(u_i^j)$	$f(u_\eta^j)$
η is even	$1 \leq \dot{l} \leq \eta - 1$	$s + [(\dot{l} - 1)(m + 1)]d$	-	-
	$\dot{l} = \eta$	$s + [(\dot{l} - 1)(m + 1) + 1]d$		-
	$1 \leq \dot{l} \leq \eta - 1$, $1 \leq j \leq m$	-	$s + [(\dot{l} - 1)(m + 1) + j]d$	-

	$1 \leq j \leq m$	-	-	$s + [(\eta - 1)(m + 1) + j]d$
η is odd	$1 \leq i \leq \eta$	$s + [(i - 1)(m + 1)]d$	-	
	$1 \leq i \leq \eta$ $1 \leq j \leq m$	-	$s + [(i - 1)(m + 1) + j]d$	-

Table 3: Labeling of vertices of the graph $C_\eta \odot K_{1,m}$

Labeling of edges				
Cases	Value of i	$g(v_i v_{i+1})$	$g(v_1 v_i)$	$g(v_i u_i^j)$
η is even	$1 \leq i \leq \eta - 1$	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_{i+1}))$	-	-
	$i = \eta$	-	$2s + 2(\|E\| - 1)d - (f(v_1) + f(v_i))$	-
	$1 \leq i \leq \eta$, $1 \leq j \leq m$	-	-	$2s + 2(\|E\| - 1)d - (f(v_i) + f(u_i^j))$
η is odd	$1 \leq i \leq \eta$, $1 \leq j \leq m$	-	-	$2s + 2(\|E\| - 1)d - (f(v_i) + f(u_i^j))$
	$1 \leq i \leq \eta - 1$	$2s + 2(\|E\| - 1)d - (f(v_i) + f(v_{i+1}))$	-	-
	$i = \eta$	-	$2s + 2(\|E\| - 1) - (f(v_1) + f(v_i))$	-

Table 4: Labeling of edges of the graph $C_\eta \odot K_{1,m}$

Therefore $f(v_i)+f(u_i^j)$ $+g(v_iu_i^j)$, $f(v_i)+f(v_{i+1})+g(v_iv_{i+1})$,$f(v_1)+f(v_i)+g(v_1v_i)$are constant equals to $2(s+(|E|-1)d)$ .Hence the $C_\eta \odot K_{1,m}$ graph admits (s,d) magic labeling.

Theorem 3.3 The η-Sunlet graph S_η is a (s,d) magic labeling.

Proof : Let G be a S_η graph. $V(S_\eta)=\{u_i\}\cup\{v_i\}$; $1\le i\le\eta$ and $E(S_\eta)=\{(u_iv_i); 1\le i\le$ η$\}\cup$ $(v_iv_{i+1}); 1\le i\le$ η-1$\}\cup\{v_1v_i); i=$η$\}$.

Define the function f from the vertex set to $\{s, s+d, s+2d \dots . s+(|E|+1)\}$, $g: E(G)\rightarrow$ $\{d, 2d, 3d \dots . .2(|E|-1)d\}$

Labeling of vertices					
Cases	Value of i	$f(u_i)$	$f(u_{i+1})$	$f(v_i)$	$f(v_{i+1})$
η is odd	$i=0$	-	s	-	$s+d$
	$1\le i \le \eta-1$	-	$s+2id$	-	$s+(2i+1)d$
η is even	$i=0$	-	s	-	$s+d$
	$i=\eta$	$s+(2(\eta-1)+1)d$	-	$s+(2(\eta-1))d$	-
	$1\le i \le \eta-2$	-	−	-	$s+(2i+1)d$

Table 5: Labeling of vertices of the graph S_η

Labeling of edges				
Cases	Value of i	$g(u_iv_i)$	$g(v_iv_{i+1})$	$g(v_1v_i)$
η is odd	$1\le i\le\eta$	$2s+2(\|E\|-1)d-\big(f(u_i)+f(v_i)\big)$	-	
	$1\le i\le \eta-1$	-	$2s+2(\|E\|-1)d-(f(v_i)+f(v_{i+1}))$	
	$i=\eta$	-	-	$2s+2(\|E\|-1)d-(f(v_1)+f(v_i))$
η is even	$1\le i\le\eta$	$2s+2(\|E\|-1)d-\big(f(u_i)+f(v_i)\big)$	-	-
	$1\le i \le \eta-1$	-	$2s+2(\|E\|-1)d-(f(v_i)+f(v_{i+1}))$	-

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	$i = \eta$		-	$2s + 2(\|E\| - 1)d - (f(v_1) + f(v_i))$

Table 6: Labeling of edges of the graph S_η

Therefore$f(u_i) + f(v_i) + g(u_i v_i)$,$f(u_i) + f(u_{i+1}) + g(u_i u_{i+1})$ are constant equals to $2(s + (|E| - 1)d)$.Hence the η-sunlet graph S_η admits (s, d) magic labeling.

Theorem 3.4 The friendship graph F_η is a (s, d) magic labeling for η≥ 3.

Proof: Let F_η be a friendship graph. V (F_η)={$\{u_i\}; 1 \leq i \leq \eta\} \cup \{v_i\}; 1 \leq i \leq \eta\} \cup \{v'\}$

E (F_η)={$(u_i v_i); 1 \leq i \leq \eta\} \cup \{(v' u_i); 1 \leq i \leq \eta\} \cup (v' v_i); 1 \leq i \leq \eta$

Here $|V(F_\eta)|$=$2\eta + 1$,$|E(F_\eta)|$=3η .Define the function f from the vertex set to

$\{s, s + d, s + 2d \ldots . s + (|E| + 1)\}$, $g: E(G) \rightarrow \{d, 2d, 3d \ldots . . 2(|E| - 1)d\ \}$

Labeling of vertices			
$f(v')$=$u_{\eta-1} + \eta d$			
Value i	$f(u_i)$	$f(u_{i+1})$	$f(v_i)$
$i = 1$	$s + (\eta + 1)d$	-	-
$i = \eta$	$s + \eta d$	-	s
$1 \leq i \leq \eta - 1$	-	-	$s + id$
$1 \leq i \leq \eta - 2$	-	$u_i + d$	-

Table 7: Labeling of vertices of the graph F_η

Labeling of edges			
Value i	$g(u_i v_i)$	$g(v' u_i)$	$g(v' v_i)$
$1 \leq i \leq \eta$	$2s + 2(\|E\| - 1)d - (f(u_i) + f(v_i))$	$2s + 2(\|E\| - 1)d - (f(v') + f(u_i))$	$2s + 2(\|E\| - 1)d - (f(v') + f(v_i))$

Table 8: Labeling of edges of the graph F_η

Therefore $f(u_i) + f(v_i) + g(u_i v_i)$,$f(v') + f(u_i) + g(v' u_i)$ and $f(v') + f(v_i) + g(v' v_i)$ are constant equals to $2(s + (|E| - 1)d)$.Hence the friendship graph F_η admits (s, d) magic labeling.

Theorem 3.5 The Flower graph Fl_η is a (s, d) magic labeling.

Proof: Let G be a graph Fl_η

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$|V(G)|$=$2\eta + 1$, $|E(G)|$=4η

$V(G)$= $\{u_i; 1 \le \dot{\iota} \le \eta\} \cup \{v_i\ ; 1 \le \dot{\iota} \le \eta\} \cup \{v\}$ and $E(G)$= $\{u_i u_{i+1}$:1≤ i ≤η-1$\} \cup \{u_1 u_\eta\} \cup$
$\{u_i v_i; 1 \le \dot{\iota} \le \eta\} \cup \{v u_i; \le \dot{\iota} \le \eta\} \cup \{v v_i; \le \dot{\iota} \le \eta\}$

Define the function f from the vertex set to $\{\{s, s + d, s + 2d \ldots . s + (|E| + 1)\},\ g: E(G) \rightarrow$
$\{d, 2d, 3d \ldots . .2(|E| - 1)d \quad \}$

Labeling of vertices					
Cases	Value of $\dot{\iota}$	$f(u_i)$	$f(v_i)$	$f(v_{i+1})$	$f(u_{i+1})$
η is odd	f(v)=s+(4η-2)d				
	$\dot{\iota} = 0$	-	-	s	$s + d$
	$1 \le \dot{\iota} \le \eta - 1$	-	-	$s + 2\dot{\iota}d$	$s + (2\dot{\iota} + 1)d$
η is even	$f(v) = s + (\eta - 2)d$				
	$\dot{\iota} = 0$	-	-	s	$s + d$
	$1 \le \dot{\iota} \le \eta - 2$		-	$s + 2\dot{\iota}d$	$s + (2\dot{\iota} + 1)d$
	$\dot{\iota} = \eta$	$u_{\eta-1} + d$	$v_{\eta-1} + 3d$	-	-

Table 9: Labeling of vertices of the graph Fl_η

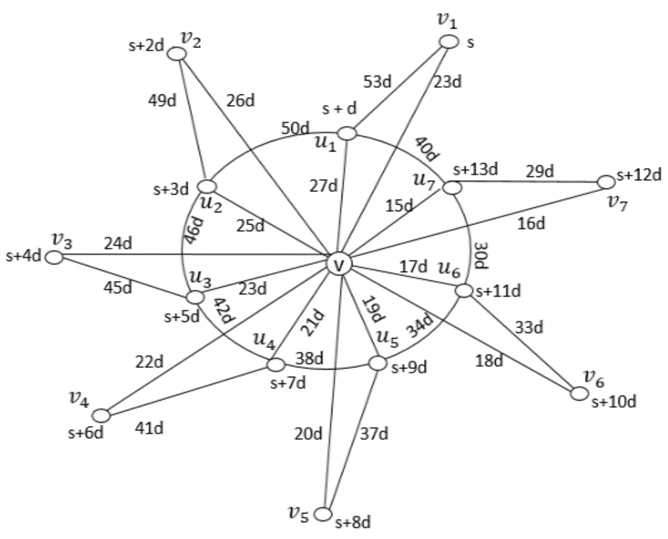
Labeling of edges						
Cases	Value of $\dot{\iota}$	$g(u_i u_{i+1})$	$g(u_i v_i)$	$g(u_i u_1)$	$g(v u_i)$	$g(v v_i)$
η is odd	$1 \le \dot{\iota} \le \eta - 1$	$2s + 2(\lvert E\rvert - 1)d - (f(u_i) + f(u_{i+1})$	-	-	-	-
	$1 \le \dot{\iota} \le \eta$	-	$2s + 2(\lvert E\rvert - 1)d - \big(f(u_i) + f(v_i)\big)$	-	$2s + 2(\lvert E\rvert - 1)d - \big(f(v) + f(u_i)\big)$	$2s + 2(\lvert E\rvert - 1)d - \big(f(v) + f(v_i)\big)$
	$\dot{\iota} = \eta$	-	-	$2s + 2(\lvert E\rvert - 1)d - \big(f(u_i) + f(u_1)\big)$	-	-

η is even	$1 \le i \le \eta - 1$	$2s + 2(\lvert E\rvert - 1)d - (f(u_i) + f(u_{i+1}))$	-	-	-	-
	$1 \le i \le \eta$	-	$2s + 2(\lvert E\rvert - 1)d - \left(f(u_i) + f(v_i)\right)$	-	$2s + 2(\lvert E\rvert - 1)d - \left(f(v) + f(u_i)\right)$	$2s + 2(\lvert E\rvert - 1)d - \left(f(v) + f(v_i)\right)$
	$i = \eta$	-	-	$2s + 2(\lvert E\rvert - 1)d - \left(f(u_i) + f(u_1)\right)$	-	-

Table 10 Labeling of Edges of the graph Fl_η

Therefore $f(u_i) + f(v_i) + g(u_i v_i)$, $f(u_i) + f(u_{i+1}) + g(u_i u_{i+1})$, $(f(u_i) + f(u_1) + g(u_i u_1)$, $f(v) + f(u_i) + g(v u_i)$ and $f(v) + f(v_i) + g(v v_i)$ are constant equals to $2(s + (\lvert E\rvert - 1)d)$.Hence the Flower graph Fl_η admits (s, d) magic labeling.

Figure1 : Flower graph Fl_7



Theorem 3.6 The Wheel graph W_η is a (s, d) magic labeling η≥ 4

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Proof : Let G = W_η . $|V(W_\eta)|$=$\eta+1$, $|E(W_\eta)|$=2η.

Define the function f from the vertex set to $\{\{s, s+d, s+2d \ldots . s+(|E|+1)\},\ g: E(G) \rightarrow \{d, 2d, 3d \ldots . .2(|E|-1)d\ \}$

Labeling of vertices			
Cases	Value of i	f (v_i)	f$(v_{\eta+1-i})$
η is odd	$1 \leq i \leq \eta$	$s+(i-1)d$	-
	$i\ =\ \eta+1$	$s+2(\eta-1)d$	-
η is even	$i=1$	s	-
	$i=\eta+1$	$s+2(\eta-1)d$	-
	$2 \leq i \leq \frac{\eta+2}{2}$	-	$s+(2i-3)d$
	$1 \leq i \leq \frac{\eta-2}{2}$	$s+2id$	-

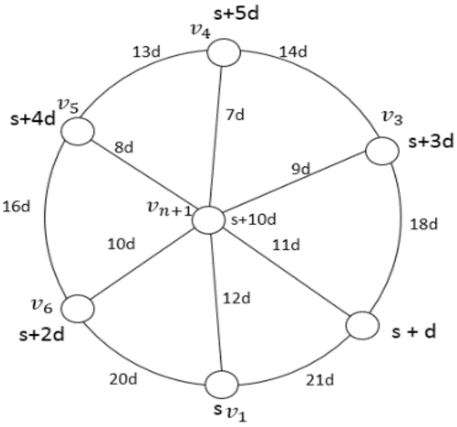
Table 11: Labeling of vertices of the graph W_η

Labeling of Edges				
Cases	Value of i	$g(v_i v_{i+1})$	$g(v_i v_1)$	$g(v_{\eta+1} v_i)$
η is odd	$i\ =\ \eta$	-	$2s+2(\|E\|-1)d - (f(v_i)+f(v_1))$	-
	$1 \leq i \leq \eta$	-	-	$2s+2(\|E\|-1)d - (f(v_{\eta+1})+f(v_i))$
	$1 \leq i \leq \eta-1$	$2s+2(\|E\|-1)d - (f(v_i)+f(v_{i+1}))$	-	-
η is even	$1 \leq i \leq \eta-1$	$2s+2(\|E\|-1)d - (f(v_i)+f(v_{i+1}))$	-	-
	$1 \leq i \leq \eta$	-	-	$2s+2(\|E\|-1)d - (f(v_{\eta+1})+f(v_i))$
	$i\ =\ \eta$	-	$2s+2(\|E\|-1)d - (f(v_i)+f(v_1))$	-

Table 12: Labeling of edges of the graph W_η

Therefore$f(v_i)+f(v_{i+1})+g(v_i v_{i+1}), f(v_i)+f(v_1)+g(v_i v_1)andf(v_{\eta+1})+f(v_i)+g(v_{\eta+1} v_i)$ are constant equals to $2(s+(|E|-1)d)$. Hence the wheel graph W_η admits (s, d) magic labeling.

Figure 2 : wheel graph W_6



4. Conclusion

The discovery of the (s,d) Magic Labeling for diverse cycle graphs underscores the versatility and applicability of this labeling scheme across different graph structures. This finding not only expands our understanding of labeling techniques but also opens up possibilities for further exploration in graph theory and its applications. Moreover, the ability to apply this labeling to various cycle graphs suggests potential advancements in areas such as network design, communication systems, and algorithm development. Further research could focus on refining the (s,d) Magic Labeling method or exploring its implications in other graph theoretic contexts. Future research will examine the (s, d) Magic labeling of additional graphs and some graph families.

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