Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 31 No. 6s (2024)

On Beta Generalized Star Pre-I-Closed Sets in Ideal Topological Spaces

S. Gowri* and V. Pankajam**
Research Scholar* and Assistant Professor**
Sri GVG Visalakshi College for Women, Udumalpet, India.
gowrisivakumar95@gmail.com and pankajamgurusamy@gmail.com

Article History:
Received: *28-05-2024*
Revised: *20-07-2024*
Accepted: *31-07-2024*

Abstract:

The purpose of this paper is to define the new idea Beta generalized star pre-closed sets, a new class of closed and open sets in topological spaces, and to examine some of its characteristics using few examples. In addition, we define Beta generalized star pre-I-closed sets, a new class of closed and open sets in ideal topological spaces and discuss through their characteristics.

Keywords: Topological spaces, pre-closed set, g*-open set, β-closed set, βg*p-closed set, βg*p-open set, βg*p-I-closed set, βg*p-I-open set.

1. Introduction

N. Levine [11] proposed the theory of generalized closed sets and generalized open sets in topological spaces. A new class of generalized pre regular closed sets in topological spaces was presented by Y. Gnanambal [4] in 1997. Beta generalized closed sets were first introduced in 2022 by Kavitha and Sasikala [9]. The notion of βg*-closed sets in topological spaces was originated by Dhanapakyam and Indirani [3]. In general topology, the idea of generalized closed sets is crucial. Numerous research articles analyzing plenty of generalized closed sets were produced following the arrival of these sets.

A non-empty set of X subsets that is closed with respect to finite union is called an ideal, I. It is recognized as an ideal space since (X, τ, I) is an ideal topological space. The local function of A for a subset A of X is given by $A^* = \{x \in X: U \cap A \notin I \text{ for each } U \in \tau(x)\}$, where τ(x) is the set of all non-empty open sets where X occurs. To avoid any confusion, just write A* from this instead of A*(I). A Kuratowski closure operator cl*(.) for a topology τ*(I, τ) is established $cl^*(A) = A \cup A^*$, which is finer than τ. Whenever A is contained in X, A's closure and interior in (X, τ) are indicated by cl(A) and int(A), respectively and A's closure and interior in (X*, τ) are shown by cl*(A) and int*(A). In this work, the concept of βg*p-closed sets in topological spaces and βg*p-I-closed sets in ideal topological spaces were presented and examined.

2. Preliminaries

Definition 2.1 [1,2] In topological space X, a subset A is termed as

i. It is semi-closed if $int(cl(A)) \subseteq A$ and semi-open if $A \subseteq cl(int(A))$.
ii. pre-closed if $cl(int(A)) \subseteq A$ and pre-open if $A \subseteq int(cl(A))$.
iii. α-closed if $cl(int(cl(A))) \subseteq A$ and α-open if $A \subseteq int(cl(int(A)))$.
iv. If A is regular-closed $A = cl(int(A))$ and if regular-open $A = int(cl(A))$.

v. β-closed(semi-pre-closed) if int(cl(int(A))) ⊆ A and β-open(semi-pre-open) if A ⊆ cl(int(cl(A))).

Definition 2.2 [5,6,17,18,19] A topological space (X, τ) subset A is referred to as

i. If cl(A) ⊆ U whenever A is a subset of U and U is open in the space X, then the set is generalized closed (g-closed) set.
ii. When A is a subset of U and U is semi open in the space X, then scl(A) ⊆ U denotes a semi generalized closed (sg-closed) set.
iii. When A is a subset of U and U is open in the space X, then scl(A) ⊆ U denotes a generalized semi closed (gs-closed) set.
iv. If αcl(A) ⊆ U whenever A is a subset of U and U is open in the space X, then the set is α generalized closed (αg-closed) set.
v. If αcl(A) ⊆ U whenever A is a subset of U and U is α-open in the space X, then the set is generalized α-closed (gα-closed) set.
vi. When A is a subset of U and U is open in the space X, then spcl(A) ⊆ U denotes a generalized semi-pre-closed (gsp-closed) set.
vii. When A is a subset of U and U is regular open in the space X, then pcl(A) ⊆ U denotes a generalized pre-regular-closed (gpr-closed) set.
viii. If cl(A) ⊆ U whenever A is a subset of U and U is regular open in the space X, then the set is regular generalized closed (rg-closed) set.
ix. If cl(int(A)) ⊆ U whenever A is a subset of U and U is open in the space X, then the set is weakly generalized (wg-closed) set.
x. A strongly generalized closed (g*-closed) set if cl(A) ⊆ U whenever A is a subset of U and U is g-open in the space X.
xi. When A is a subset of U and U is g-open in the space X, then (cl(int(A)) ⊆ U denotes a mildly generalized closed (mildly g-closed) set.
xii. When A is a subset of U and U is g-open in the space X, then pcl(A) ⊆ U denotes a generalized star pre-closed (g*p-closed) set.
xiii. If βcl(A) ⊆ U whenever A is a subset of U and U is g-open in the space X, then the set is beta generalized closed (βg-closed) set.
xiv. If cl(int(A)) ⊆ U whenever A is a subset of U and U is g-open in the space X, then the set is beta star closed (β*-closed) set.
xv. If gcl(A) ⊆ U whenever A is a subset of U and U is β-open in the space X, then the set is beta generalized star closed (βg*-closed) set.
xvi. If pcl(A) ⊆ U whenever A is a subset of U and U is β-open in the space X, then the set is beta generalized pre-closed (βgp-closed) set.

Definition 2.3 Let be a topological space (X, τ). Let I represent an ideal on X. When the space (X, τ, I) satisfies the two requirements, it is referred to as an ideal topological space,

i. If A ∈ I and B ⊆ A ⇒ B ∈ I.
ii. If A ∈ I and B ∈ I, then A ∪ B ∈ I.

Definition 2.4 [16] An Ideal topological space (X, τ, I) subset A is referred to as

i. If cl*(int(A)) ⊆ A, then the pre-I-closed set. A is referred to as a pre-I-open if A ⊆ (int(cl*(A)).
ii. If int(cl*(A)) ⊆ A then the semi-I-closed set. A is referred to as a semi-I-open if A ⊆ (cl*(int(A)).
iii. If cl*(int(cl*(A))) ⊆ A then α-I-closed set. A is referred to as a α-I-open if A ⊆ (int(cl*(int(A))).
iv. The closed set β-I is if (int(cl*(int(A))) ⊆ A. A is referred to be a β-I-open if A ⊆ (cl*(int(cl*(A))).
v. If A = cl*(int(A)) then the set is regular-I-closed. A set is considered regular-I-open if A = (int(cl*(A)).

Lemma 2.5 [12] Let X has two subsets A and B. An ideal topological space is (X, τ, I). The preceding characteristics are then:

i. A ⊆ B ⇒ A* ⊆ B*,
ii. A* = cl(A*) = cl(A) = cl*(A),
iii. (A ∪ B) * = A* ∪ B*,
iv. (A ∩ B) * ⊆ A* ∩ B*,
v. (A*) * ⊆ A*.

3. βg*p-Closed Sets in Topological Spaces

Definition 3.1 If pcl(B) ⊆ D whenever B is a subset of D (B ⊆ D) and D is g*-open in X, then subset B of a topological space (X, τ) is called Beta Generalized Star Pre-Closed Set (briefly βg*p-closed).

Theorem 3.2 Each closed set is closed in βg*p-closed.

Proof: In the topological spaces, let B represent any closed set. Let D be any open set g* that contains B. pcl(B) equals B. Since B is a closed set. Consequently, pcl(B) ⊆ D. B is therefore βg*p-closed in X. The following example demonstrates why the converse of the preceding theorem need not be true.

Example 3.3 Assume that the set X = {u, v, w}, considering a topology τ = {ϕ, {v}, {u, w}, X} and its closed form τ^c = {ϕ, {u, w}, {v}, X}. The closed sets of βg*p are {ϕ, {u}, {v}, {w}, {u, v}, {v, w}, {u, w}, X}. In this case, B = {w} is a closed set of βg*p but not a closed set.

Theorem 3.4 Each and every βg*-closed set is a βg*p-closed set.

Proof: Suppose that D is any g*-open set in X such that B ⊆ D and that B is a βg*-closed set in the space (X, τ). Assume that each g*-open set is β-open. Considering the closure of B, pcl(B) ⊆ cl(B) ⊆ D. This clarifies that B is a closed set in (X, τ) with βg*p. The following example demonstrates, the converse of the aforementioned theorem need not be true.

Example 3.5 With the topology τ = {ϕ, {v, w}, X}, let X = {u, v, w}. The closed sets of βg*p form are {ϕ, {u}, {v}, {w}, {u, v}, {u, w}, X}. It is not a βg*-closed set, although in this case, B = {v} is βg*p-closed set.

Theorem 3.6 Any βg*p-closed set can also be a βg-closed.

Proof: Consider B to be a βg*p-closed set in the space (X, τ). For any g-open set which has B. Each g-open set is g*-open, accordingly pcl(B) ⊆ D. B is a closed set, so it is βg. As the following example demonstrates, the converse of the aforementioned theorem need not be true.

Example 3.7 With the topology τ = {ϕ, {u}, {v}, {v, u}, X}, let X = {u, v, w}. The closed sets of βg*p structure are {ϕ, {w}, {v, w}, {u, w}, X}. This is not a βg*p-closed set rather, B = {u} is a βg-closed set.

Theorem 3.8 In the topological space (X, τ), every weakly generalized-closed, regular weakly generalized-closed, generalized*pre-closed, mildly g-closed, β*-closed, α generalized-closed, generalized α-closed sets in the space are βg*p-closed.

Proof: As a result, each open set in the space (X, τ) is g*-open set. The theorems converse need not always be true. The example that follows exemplifies it.

Example 3.9 Assume that the set X = {u, v, w}, considering a topology τ = {ϕ, {u}, {w}, {u, v}, {u, w}, X}. The closed sets of βg*p are {ϕ, {w}, {v}, {v, w}, {u, v}, X}. Here B = {v, w} is βg*p-closed set but it is not a wg-closed and rwg-closed sets.

Example 3.10 Assume that X = {u, v, w} and its topology τ = {ϕ, {w}, X}. {ϕ, {u}, {v}, {v, w}, {w, u}, {u, v}, X} are the βg*p-closed sets of X. Although it is a βg*p-closed set in this case B = {v, w} is not a g*p-closed and mildly g-closed sets.

Example 3.11 Given X = {u, v, w} and topology τ = {ϕ, {u, v}, X}. {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, X} are the βg*p-closed sets of X. While, B = {u} is not a αg-closed and gα-closed sets it is βg*p-closed set.

Remark 3.12 The class of regular generalized-closed, generalized pre-regular closed and generalized semi pre-closed sets in the topological space (X, τ) is independent of the class of βg*p-closed sets in topological space.

Example 3.13 With the topology τ = {ϕ, {u}, {w}, {u, w}, X}, let X = {u, v, w}. The closed sets of βg*p form are {ϕ, {v}, {v, w}, {u, v}, X}. It is not a βg*p-closed set, although in this case, B = {u, w} is rg-closed and gpr-closed sets.

Example 3.14 With the topology τ = {ϕ, {u}, {v}, {v, u}, X} and let X = {u, v, w}. The closed sets βg*p form are {ϕ, {w}, {v, w}, {u, w}, X}. It is not a βg*p-closed set, although in this case, B = {v} is gsp-closed set.

Theorem 3.15 Union of two βg*p-closed sets is βg*p-closed set in X.

Proof: Take M and N as two βg*p-closed sets in X to demonstrate the proof. Assume D is any g*-open set in (X, τ) such that M ∪ N ⊆ D. Following that D contained in both M and N. As a result, M and N are βg*p-closed sets and pcl(M) ⊆ D and pcl(N) ⊆ D. For this reason, pcl(M) ∪ pcl(N) ⊆ pcl (M ∪ N) ⊆ D. Therefore, M ∪ N is a closed set of βg*p.

Example 3.16 With the topology, τ = {ϕ, {w}, {v, w}, {u, w}, X} and let X = {u, v, w}. The closed sets of βg*p = {ϕ, {v}, {u}, {u, v}, X}. Take M = {u} and N = {v} be closed sets similarly their union M ∪ N = {u, v} are also βg*p-closed set.

Remark 3.17 The intersection of two βg*p-closed sets need not be βg*p-closed set.

Example 3.18 With the topology τ = {ϕ, {w}, X} and take X = {u, v, w}. {ϕ, {u}, {v}, {u, v}, {v, w}, {w, u}, X} are the closed sets of βg*p. Let M = {u, v} and N = {v, w}. Similarly, their intersection M ∩ N = {v} is also in βg*p-closed sets.

Theorem 3.19 A subset B of X is βg*p-closed set in (X, τ) if and only if B ⊆ C ⊆ pcl(B).

Proof: Assume that D is a g*-open set in X with C ⊆ D. pcl(B) ⊆ D, since B ⊆ D and B is βg*p-closed. We have pcl(C) ⊆ pcl(pcl(B)) = pcl(B) as C ⊆ pcl(B). Consequently, pcl(C) ⊆ D. Hence, C is a closed set of (X, τ) that is βg*p.

Theorem 3.20 In (X, τ), B is βg*p-closed if it is both open and βg-open.

Proof: Let B ⊆ D and D be g*-open. At this point B ⊆ B. According to the hypothesis, βcl(B) ⊆ B. Every β-closed set is pre-closed, therefore pcl(B) ⊆ cl(B). As a result, pcl(B) ⊆ B ⊆ U. Therefore, B is βg*p-closed set.

Theorem 3.21 If B ⊆ X is a βg*p-closed set, then there is no non-empty β-closed set in pcl(B)-B.

Proof: Assume that B is a βg*p-closed set and C is a closed set in X such that C ⊆ pcl(B) - B. Then C ⊆ pcl(B) and C ⊆ X - B implies B ⊆ X - C. Given that B is a βg*p-closed set and X - C is a β-open set containing B, it can be proved that pcl(B) ⊆ X - C and thus C ⊆ X-pcl(B). As a result, it follows that C = ϕ, C - pcl(B) = ϕ.

4. βg*p-Open Sets in Topological Spaces

Definition 4.1 If the complement of a subset B of a topological space (X, τ) is βg*p-closed, then the subset is referred to as a Beta Generalized Star Pre (βg*p)-Open Set.

Theorem 4.2 When C is g*-closed in X and C ⊆ B, a subset B of a topological space (X, τ) is βg*p-open if and only if C ⊆ pint(B).

Proof: Necessity: Assume C is g*-closed in (X, τ) and C ⊆ B. Let C ⊆ pint(B) where E is g*-open, let B^c ⊆ E. Therefore, E^c is g*-closed in E^c ⊆ B. Accordingly, E^c ⊆ pint(B) by assumption which implies $(pint(B))^c$ ⊆ E. Consequently, pcl(B^c ⊆ E). Since B is implied to be βg*p-open, B^c is βg*p-closed.

Sufficiency: Let F ⊆ B, where F is g*-closed and let B be βg*p-open in X. With B^c ⊆ F^c, where F^c is g*-open, we have B^c is βg*p-closed. pcl (B^c) ⊆ F^c provides F ⊆ X - pcl(B^c) = pint(X – B^c) = pint(B).

Theorem 4.3 If B is a βg*p-open subsets of (X, τ) and pint(B) ⊆ C ⊆ B, then B is moreover a βg*p-open subset of (X, τ).

Proof: Given that pint(B) ⊆ C ⊆ B, B^c ⊆ C^c ⊆ pcl(B^c). With B^c being βg*p-closed. C^c is βg*p-closed by theorem 3.19. Consequently, C is βg*p-open.

Theorem 4.4 If B ⊆ X is βg*p-closed then pcl(B) -B is g*-open.

Proof: Let B be a closed set in X that is βg*p. Given a g*-closed set E, assume that E ⊆ pcl(B) - B. Consequently, there is not a single non-empty g*-closed set in pcl(B) - B. Therefore E = ϕ and E ⊆ int(pcl(B) - B). As can be shown, pcl(B) - B is g*-open. B is consequently g*p-closed in (X, τ).

5. βg*p-I-Closed Sets in Ideal Topological Spaces

Definition 5.1 An ideal topological space (X, τ, I) has a subset B known as the Beta generalized star pre-I-closed set if B* ⊆ D whenever B ⊆ D and D is β*-open.

Theorem 5.2 A closed set is always βg*p-I-closed.

Proof: Let D be a β*-open set in the space (X, τ, I) that contains B*. Let B be closed subset of the ideal topological space. B = B* is what we obtain as the impact of B's closure. As a result, B* ⊆ cl*(B) ⊆ D. This suggest a βg*p-I-closed set, designated as B. Not every time the preceding theorems converse true.

Example 5.3 Construct a topology, τ = {ϕ, X} and set X = {u, v, w}, I = {ϕ, {u}, {w}, {u, w}}. The closed sets consisting of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, {u, v}, X}. B = {w} is a closed set in terms of βg*p-I, despite it is not a closed set in this particular case.

Theorem 5.4 A set which is always βg*p-I-closed is βgp-closed.

Proof: Let D be a β*-open set in the space (X, τ, I) that contains B*. Let B be the ideal topological space's closed set. Since B's closure has an impact, we obtain B = B*. As a consequence, B* ⊆ cl*(B) ⊆ D. This indicates that B is a βg*p-I-closed set. Not every time is the preceding theorem's converse is true.

Example 5.5 Create a topology, τ = {ϕ, {u, w}, {v, w}, {w}, X} and set X = {u, v, w}, I = {ϕ, {w}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, {u, v}, X}. In this case, B = {w} is a closed set in terms of βg*p-I, even though it is not a βgp-closed set.

Theorem 5.6 All βg*p-closed sets are βg*p-I-closed.

Proof: Let B be a βg*p-I-closed set in the ideal topological space (X, τ, I). Allow D to be an open set in the space such that B* ⊆ D. For every open set to be β*-open, B must be βg*p-I-closed. But pcl(B) ⊆ B* is a constant. Hence, pcl(B) ⊆ D. B is thus a βg*p-closed set in ideal topological space. The inverse of this theorem generally proves true.

Example 5.7 Determine a topology, τ = {ϕ, {u, v}, {u, w}, {w}, {u}, X} and set X = {u, v, w}, I = {ϕ, {u}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {u, v}, X}. In this case, βg*p-closed in set B = {w, u} but βg*p-I is not closed.

Theorem 5.8 A pre-closed, α generalized-closed, g*-closed, weakly generalized-closed sets are equivalent to a βg*p-I-closed set in the ideal topological space.

Proof: Every open set in the space is a β*-open. The following examples show that the contradiction in the preceding theorem need not hold.

Example 5.9 Consider a topology, τ = {ϕ, {u, v}, {v, w}, {v}, X} and set X = {u, v, w}, I = {ϕ, {v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, {u, v}, X}. In this case, B = {w, v} is a closed set in terms of βg*p-I, even though it is not a pre-closed set.

Example 5.10 Construct a topology, τ = {ϕ, {u}, {v}, {u, v}, {u, w}, X} and set X = {u, v, w}, I = {ϕ, {u}, {v}, {u, v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {w, u}, {u, v}, X}. In this case, B = {u, v} is a closed set in terms of βg*p-I, even though it is not a α generalized-closed set.

Example 5.11 Determine a topology, τ = {ϕ, {u, v}, {v, w}, {w}, {v}, X} and set X = {u, v, w}, I = {ϕ, {v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {w, u}, {u, v}, X}. In this case, B = {v} is a closed set in terms of βg*p-I, even though it is not a g*-closed set.

Example 5.12 Set X = {u, v, w}, I = {ϕ, {v}} and topology, τ = {ϕ, {u, v}, {v, w}, {u}, {v}, X}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, X}. In this case, B = {w, u} is βg*p-I-closed set but it is not a weakly generalized-closed set.

Remark 5.13 A generalized α-closed, generalized star pre-closed, mildly g-closed, generalized semi pre-closed, regular generalized-closed, generalized pre-regular closed, regular weakly generalized-closed, semi generalized-closed, generalized semi-closed, beta generalized-closed and beta generalized star-closed sets are all considered to be beyond of the boundaries of the concept of βg*p-I-closed sets in the space.

Example 5.14 Take a topology, τ = {ϕ, {u, w}, {v, u}, {u}, {v}, X} and set X = {u, v, w}, I = {ϕ, {u}, {v}, {u, v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {w, u}, {u, v}, X}. In this case, generalized α are closed in set B = {v, w} but not a βg*p-I-closed set.

Example 5.15 Construct a topology, τ = {ϕ, {u, w}, X} and set X = {u, v, w}, I = {ϕ, {w}, {v}, {w, v}}. The closed sets consist of βg*p-I-closed = {ϕ, {v}, {w}, {v, w}, X}. In this case, g*p and mildly g are closed in set B = {u, v} but not a βg*p-I-closed set.

Example 5.16 Define a topology, τ = {ϕ, {u, v}, {v, w}, {v}, {w}, X} and set X = {u, v, w}, I = {ϕ, {v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {w, u}, {u, v}, X}. In this case, regular generalized are closed in set B = {v, w} but not a βg*p-I-closed set.

Example 5.17 Consider a topology, τ = {ϕ, {u}, {v, w}, X} and set X = {u, v, w}, I = {ϕ, {w}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {w}, {v, w}, X}. In this case, regular weakly generalized and generalized semi pre are closed in set B = {v} but not a βg*p-I-closed set.

Example 5.18 Determine a topology, τ = {ϕ, {u, v}, {v, w}, {u}, {v}, X} and set X = {u, v, w}, I = {ϕ, {v}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {w}, {v, w}, {w, u}, X}. In this case, generalized pre regular are closed in set B = {u, v} but not a βg*p-I-closed set.

Example 5.19 Create a topology, τ = {ϕ, {u, w}, {u}, {w}, X} and set X = {u, v, w}, I = {ϕ, {u}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {v}, {v, w}, {u, v}, X}. In this case, semi generalized and generalized semi are closed in set B = {w} but not a βg*p-I-closed set.

Example 5.20 Extract a topology, τ = {ϕ, {u, v}, {w}, X} and set X = {u, v, w}, I = {ϕ, {w}}. The closed sets consist of βg*p-I-closed = {ϕ, {u}, {w}, {u, v}, X}. In this case, β generalized and β generalized* are closed in set B = {u, w} but not a βg*p-I-closed set.

Theorem 5.21 Two closed sets βg*p-I are united in any ideal topological space (X, τ, I).

Proof: Suppose that M and N are two closed sets of βg*p-I in the space (X, τ, I). Assume that any β*-open set D in X such that M ∪ N ⊆ D. Since M ⊆ D and N ⊆ D, M and N are βg*p-I-closed sets. Being that D is β*-open whenever M* ∪ N* = (M ∪ N) * ⊆ D, M* ⊆ D and N* ⊆ D. In the ideal topological space, M ∪ N is a closed set with respect to βg*p-I.

Example 5.22 Set X = {u, v, w}, I = {ϕ, {u}, {w}, {u, w}} and topology, τ = {ϕ, X}. The closed sets include βg*p-I = {ϕ, {u}, {v}, {w}, {u, v}, {v, w}, {w, u}, X}. Given two closed sets of βg*p-I, let M = {v, w} and N = {u}, then their union M ∪ N = {X} is a closed set of βg*p-I.

Remark 5.23 There is no requirement for the intersection of two βg*p-I-closed sets.

Theorem 5.24 A subset B of a topological space X is a g*p-I-closed set in the topological space (X, τ, I) if it is both pre-open and β-closed.

Proof: Let B be a pre-open and β-closed set in the space (X, τ, I). Hypothesize that D is a β*-open set in the space (X, τ, I) and suppose that B ⊆ D. Considering that B ⊆ B, B* ⊆ B and then B* ⊆ B ⊆ D. In consequence, B is βg*p-I-closed set in the ideal topological space. If B is pre-open and βg*p-I-closed sets in the space (X, τ, I), then it is need to be a β-closed set as demonstrated by the example below.

Example 5.25 Take a topology, τ = {ϕ, {u}, {v}, {w}, {u, v}, {v, w}, X}, X = {u, v, w} and I = {ϕ, {v}}. The closed sets of βg*p-I = {ϕ, {u}, {v}, {w}, {u, v}, {v, w}, {w, u}, X}. β = {ϕ, {u}, {w}, {u, w}, X} are the closed sets of X. From this, it is possible to seen that B = {v} is pre-open and βg*p-I-closed sets, but not a β-closed set.

Theorem 5.26 In the ideal topological space (X, τ, I), if B is *β-open and βg*p-I-closed, then B is g*p-closed.

Proof: B ⊆ B, since B is both β*-open and βg*p-I-closed, B* ⊆ B. Consequently, B* = B. B has become an g*p-closed set in X.

Theorem 5.27 In an ideal topological space (X, τ, I), let B be a βg*p-I-closed such that B ⊆ C B*. Thus, B is also a closed set in βg*p-I.

Proof: Let D be an β*-open set which contains C. B ⊆ C ⊆ D ⇒ C* ⊆ B* ⊆ D. Therefore, B is closed in βg*p-I.

6. βg*p-I-Open Sets in Ideal Topological Spaces

Definition 6.1 An βg*p-I-open set is defined as the complement of a βg*p-I-closed set.

Theorem 6.2 If M and N are βg*p-I-open sets of the ideal topological space (X, τ, I) then, M ∩ N also βg*p-I-open set.

Proof: Assuming two βg*p-I-open sets M* and N* in X. Considering that $(M^*)^c$ and $(N^*)^c$ are closed sets of βg*p-I in X. $(M^*)^c \cup (N^*)^c$ is a βg*p-I-closed in X by theorem 5.21. It indicates βg*p-I-closed in X is $(M^* \cap N^*)^c$. M ∩ N is therefore an open set in X with βg*p-I.

Theorem 6.3 C* is βg*p-I-open in X if int*(C) ⊆ C* ⊆ B* and if B* is βg*p-I-open in X.

Proof: Assume that if B* is βg*p-I-open in X and int*(C) ⊆ C* ⊆ B, then $(B^*)^c \subseteq (C^*)^c \subseteq cl^*(B)^c$. By theorem 6.2, C* is βg*p-I-open in X since $(B^*)^c$ is βg*p-I-open in X.

Theorem 6.4 When G is a g-closed set and G ⊆ B*, a subset B* is β*-open if and only if G ⊆ pint*(B).

Proof: Essential: Assume that B is an open set along with G ⊆ B* is a g-closed subset of (X, τ, I). By definition, the set β*-closed contains X - B*. Furthermore, the g-open set X - G contains X - B*. βcl*(X

- B) ⊆ X - G is implied X - βint*(B) = βcl*(X - G) at this point. Thus, X - βint*(B) ⊆ X - G or G ⊆ pint*(B) follows.

Sufficiency: In the instance that G is a g-closed set and G ⊆ βint*(B) where G ⊆ B, X - B ⊆ X - G and βint*(B) ⊆ X - G follows that βcl*(X - B) ⊆ X - G. Thus, X - B is a β*-closed set and B turns into a β*-open set.

References

[1] P. Bhattacharyya and B. K. Lahiri, Semi generalized closed sets in topology, Indian J. Math., 29(1987), 376-382.

[2] P. Bhattacharya, On Generalized regular closed sets, Int. J, Contemp. Math. Sciences., 6 (2011), 145-152.

[3] C. Dhanapakyam and K. Indirani, On βg*-closed sets in topological spaces, International journal of applied research., 2(4) (2016), 388-391.

[4] Y. Gnanambal, On generalized pre regular closed sets in topological spaces, Indian J. Pure Appl. Math., 28(1997), 351-360.

[5] S. Gowri and V. Pankajam, On Beta generalized pre closed sets in topological spaces, Journal of Southwest Jiaotong University., 58(2), (2023), 559-564.

[6] S. Gowri and V. Pankajam, On Beta generalized-I-closed sets in ideal topological spaces, Indian Journal of Natural Sciences., 14(82), (2024), 67683-67688.

[7] S. Jafari, R. M. Latif, G. Nordo and N. Rajesh, Properties of α-open sets in ideal generalized topological spaces, Poincare Journal of Analysis and Applications., Vol.8, (2021), 231-244.

[8] Jose Tormet, Rafael Lozada-Yavina and Margot Salas-Brown, A generalized local function in ideal topological spaces, Poincare Journal of Analysis and Applications, Vol.9, (2022), 77-89.

[9] V. Kavitha and V. E. Sasikala, Beta Generalized closed sets in topological spaces, Journal of Algebraic Statistics., ISSN:1309-3452, Vol.13, No.3-(2022), 891-898.

[10] M. Khan and T. Noiri, On gI-closed sets in ideal topological spaces, Journal of Advanced studies in topology., Vol.1, (2010), 29-33.

[11] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo., 19(2) (1970), (89-96).

[12] S. Maragathavalli and D. Vinodhini, On α generalized closed sets in ideal topological spaces, IOSR Journal of Mathematics., Vol.10, (2014), 33-38.

[13] M. W. Mohammed and A. A. Mohammed, Some classes in ideal topological spaces, Journal of education and sciences., 32(2), (2023), 128.

[14] O. Najastad, On some classes of nearly open sets, Pacific J. Math.,15(1965), 961-970.

[15] N. Palaniappan and K. C. Rao, Regular generalized closed sets, Kyungpook Math., 3(2) (1993), 211.

[16] P. G. Palanimani and R. Parimelazhagan, β*-closed sets in topological spaces, IOSR Journal of Mathematics., Vol 5, (2013), 47-50.

[17] V. Pankajam and S. Gowri, On αωI-closed sets in ideal topological spaces, Journal of Xidian University.,15(3), (2021), 30-36.

[18] V. Pankajam and S. Gowri, On generalized star ωαI-closed sets in ideal topological spaces, Bull. Pure Appl. Sci. Sect. E Math. Stat. 40E (1), (2021), 121-126.

[19] J. K. Park, Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos Solutions and Fractals., 20-(2004), 1103-1111.

[20] M. K. R. S. Veerakumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 21(2000),1-19.